# Propensity Scores 

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## Four Learning Objectives

1. Why observational studies have little ability to make causal claims
2. Understanding the niche that observational studies fill
3. What is a propensity score
4. Ways to implement a propensity score

## Outline

1. Background on assessing causation
2. Define propensity score (PS)
3. Calculate the PS
4. Use the PS
5. Limitations of the PS

## Causality

- Researchers are often interested in understanding causal relationships
- Does drinking red wine affect health?
- Does a new treatment reduce symptoms?
- Does job burnout affect risk of suicidality?
- Does the Veterans Crisis Line reduce the likelihood of suicide?


## Randomized Clinical Trial

- RCT provides a methodological approach for understanding causation
- Understanding propensity score is assisted by understanding randomized trials.


## Randomization



Note: random sorting can, by chance, lead to unbalanced groups. Most trials use checks and balances to preserve randomization
Just because a RCT can speak to causality, you must ask the question for whom- generalizability is often very limited

## Trial analysis

- The expected effect of treatment is

$$
E(Y)=E\left(Y^{A}\right)-E\left(Y^{B}\right)
$$

Expected effect on group A minus expected effect on group B (i.e., mean difference).

## Trial Analysis (II)

- $E(Y)=E\left(Y^{A}\right)-E\left(Y^{B}\right)$ can be analyzed using the following general model

$$
y_{i}=\alpha+\beta x_{i}+\varepsilon_{i}
$$

Where

- y is the outcome
- $\alpha$ is the intercept
- x is the mean difference in the outcome between treatment A relative to treatment B
- $\varepsilon$ is the error term
- i denotes the unit of analysis (person)


## Trial Analysis (III)

- The model can be expanded to control for baseline characteristics

$$
y_{i}=\alpha+\beta x_{i}+\delta z_{i}+\varepsilon_{i}
$$

Where

- y is outcome
- $\alpha$ is the intercept
- $\quad x$ is the added value of the treatment $A$ relative to treatment $B$
- $Z$ is a vector of baseline characteristics (predetermined prior to randomization)
- $\varepsilon$ is the error term
- i denotes the unit of analysis (person)


## Assumptions

- Right hand side variables are measured without noise (i.e., considered fixed in repeated samples)
- There is no correlation between the right hand side variables and the error term $E\left(x_{i} \varepsilon_{i}\right)=0$

- If these conditions hold, $\beta$ is an unbiased estimate of the causal effect of the treatment on the outcome


## What if...

- the assumptions don't hold in the RCT, then what?
- You lose the unbiased estimate of causality.


## Observational Studies

- Randomized trials may be
- Unethical
- Infeasible
- Impractical
- Not scientifically justified


## Endogenous

- Poll-
- Has anyone heard of this term?
- Yes, I use the term frequently when talking to friends and family
- Yes, I have heard others use the term related to methods
- Yes, I have heard the term related to medicine or endocrinology
- No, and I like being honest


## Endogenous

- Not attributable to any external factor.
- Example: Does smoking lead to cancer
cancer $_{i}=\alpha+\beta$ smoking $_{i}+\varepsilon_{i}$
- Smoking is correlated with income, education, parental exposure, etc.
- We aren't controlling for any of those factors, thus E(smoking $\left.{ }_{i}, \varepsilon_{i}\right) \neq 0$
- Thus, smoking is endogenous


## Sorting without randomization



## Sorting without randomization



## Sorting without randomization



## Propensity Score Defined

- The PS uses observed information, which is multi-dimensional, to calculate a single variable (the score)
- The score is the predicted propensity to get sorted (usually thought of as propensity to get treatment).

Expected treatment effect: $E(Y)=E\left(Y^{\mathrm{A}}\right)-E\left(Y^{\mathrm{B}}\right)$
Propensity Score is: $\operatorname{Pr}\left(\mathrm{Y}=\mathrm{A} \mid \mathrm{X}_{\mathrm{i}}\right)$

## Propensity Scores

- What it is: Another way to correct for observable characteristics
- What it is not: A way to adjust for unobserved characteristics
- The only way to make causal claims is to make huge assumptions.


## Strong Ignorability

- To make statements about causation, you would need to make an assumption that treatment assignment is strongly ignorable.
- Similar to assumptions of missing at random
- Equivalent to stating that all variables of interest are observed


## Creating a <br> Propensity Score

## Calculating the Propensity Score

■ You observe treatment: One group receives it and another group doesn't
■ Use multivariate logistic regression to estimate the probability that a person received treatment

- The predicted probability from the logistic model is the propensity score


## Variables to Include

- Include variables that are related to the observed outcome
- This will decrease the variance of an estimated exposure effect without increasing bias
- Do not include variables affect only correlated with exposure



## Variables to Exclude

■ Exclude variables that are related to the exposure but not to the outcome

- These variables will increase the variance of the estimated exposure effect without decreasing bias
- Variable selection is particularly important in small studies ( $\mathrm{n}<500$ )


## Example: Resident Surgery

- Do cardiac bypass patients have better / worse outcomes when their surgery is conducted by a resident or an attending?
- We had a datasets that tracked the primary surgeon for heart bypass


## Uses

■ Understanding sorting and balance

- Sorting is multidimensional
- The PS provides a simple way of reducing this dimensionality to understand the similarity of the treatment groups
- Adjusting for covariance


## Example

- Are surgical outcomes worse when the surgeon is a resident?
- Resident assignment may depend on
- Patient risk
- Availability of resident
- Resident skill
- Local culture


## Resident Assignment

|  | OR | $P$ value |  |
| :---: | :---: | :---: | :---: |
| Age | 1.00 | 0.79 |  |
| Canadian Functional Class |  |  |  |
| Class 2 | 1.93 | 0.15 |  |
| Class 3 | 2.12 | 0.09 | Assignment not |
| Class 4 | 4.25 | 0.02 | associated with age |
| Urgent priority | 0.93 | 0.89 | or number of grafts |
| Artery condition at site |  |  |  |
| Calcified | 0.67 | 0.25 |  |
| Sclerotic | 2.63 | 0.00 |  |
| site 2 | 62.89 | 0.00 | Assignment |
| site 3 | 0.67 | 0.60 | associated with |
| site 5 | 138.16 | 0.00 | angina symptoms |
| site 7 | 11.66 | 0.00 | and planned |
| site 8 | 19.85 | 0.00 | harvesting technique |
| site 9 | 1.76 | 0.43 |  |
| endo vascular harvest | 0.20 | 0.01 |  |
| On pump surgery | 1.20 | 0.75 |  |
| 1-2 grafts | 1.70 | 0.16 |  |
| 4-5 grafts | 0.79 | 0.46 |  |

## Shared or Common Support

- Concept that measures overlap of people in both treatments

■ Conditional on covariates, there exist people who choose both treatments.

- Poor common support suggests that conditional on observables, we cannot control for sorting.


## Propensity Score for Resident vs Attending Surgeon



## Compare Three Scores



## Poll

- Do any of these distributions concern you? Choose one
- A
- B
$-\mathrm{C}$
- All of them
- None of them


## RCTs and Propensity Scores

- What would happen if you used a propensity score with data from a RCT?


## Shared Common Support



## Common Support

- Growing evidence in economics that propensity scores provide some advantages when there is considerable shared support


## Using the Propensity Score

## Using the Propensity Score

1. Compare individuals based on similar PS scores (a matched analysis)
2. Conduct subgroup analyses on similar groups (stratification)
3. Include it as a covariate (quintiles of the PS) in the regression model
4. Use it to weight the regression (i.e., place more weight on similar cases)
5. Use both 3 and 4 together (doubly robust)

## PS as a Covariate

- There seems to be little advantage to using PS over multivariate analyses in most cases. ${ }^{1}$
- PS provides flexibility in the functional form
- Propensity scores may be preferable if the sample size is small and the outcome of interest is rare. ${ }^{2}$

1. Winkelmeyer. Nephrol. Dial. Transplant 2004; 19(7): 1671-1673.
2. Cepeda et al. Am J Epidemiol 2003; 158: 280-287

## Matched Analyses

- The idea is to select controls that resemble the treatment group in all dimensions, except for treatment
- You can exclude cases and controls that don't match, which can reduce the sample size/power.
■ Different matching methods


## Matching Methods

■ Nearest Neighbor: rank the propensity score and choose control that is closest to case.

- Caliper: choose your common support and from within randomly draw controls
- Choice of matching estimator important


## Recent Areas of Research

- Economics: choice of matching estimators
- Busso M et al. New Evidence on the Finite Sample Properties of Propensity Score Reweighting and Matching Estimators. Review of Economics and Statistics, 96.5 (2014): 885-897
- Biostatistics: high dimensional propensity scores using big data
- Schneeweiss, Sebastian, et al. "High-dimensional propensity score adjustment in studies of treatment effects using health care claims data." Epidemiology 20.4 (2009): 512.


## Limitations

## Do the Unobservables Matter?

- Propensity scores focus only on observed characteristics, not on unobserved.
- Improbable that we fully observe the sorting process
- Thus $\mathrm{E}\left(\mathrm{x}_{\mathrm{i}} \varepsilon_{\mathrm{i}}\right) \neq 0$
- Multivariate (including propensity score) is biased and we need instrumental variables, fixed effects or RCT


## Does Using PS Exacerbate Imbalance of Unobservables

- PS is based on observables.
- Brooks and Ohsfeldt, using simulated data, showed that PS models can create greater imbalance among unobserved variables.

Brooks and Ohsfeldt (2013): Squeezing the balloon: propensity scores and unmeasured covariate balance. Health Services Research.

## Summary

## Overview

- Propensity scores offer another way to adjust for confound by observables
- Reducing the multidimensional nature of confounding can be helpful
- There are many ways to implement propensity scores and a growing interest in matching estimators


## Strengths

- Allow one to check for balance between control and treatment
- Without balance, average treatment effects can be very sensitive to the choice of the estimators. ${ }^{1}$

1. Imbens and Wooldridge 2007 http://www.nber.org/WNE/lect_1_match_fig.pdf

## Challenges

- Propensity scores are often misunderstood
- Not enough attention is placed on the PS model, itself
■ Not enough attention is placed on robustness checks
- While a PS can help create balance on observables, PS models do not control for unobservables or selection bias


## Further Reading

- Imbens and Wooldridge (2007) www.nber.org/WNE/lect_1_match_fig.pdf
- Imbens, Guido W. "The role of the propensity score in estimating dose-response functions." Biometrika 87.3 (2000): 706-710.
- Imbens, Guido W. "Nonparametric estimation of average treatment effects under exogeneity: A review." Review of Economics and Statistics 86.1 (2004): 4-29.
- Guo and Fraser (2010) Propensity Score Analysis. Sage.
- Rosenbaum, P. R., D. B. Rubin. "The central role of the propensity score in observational studies for causal effects." Biometrika 70 (1983): 41-55
- Brooks, John M., and Robert L. Ohsfeldt. "Squeezing the balloon: propensity scores and unmeasured covariate balance." Health Services Research 48.4 (2013): 1487-1507.
- Garrido, Melissa M., et al. "Methods for constructing and assessing propensity scores." Health Services Research 49.5 (2014): 1701-1720.
- Busso M et al. "New Evidence on the Finite Sample Properties of Propensity Score Reweighting and Matching Estimators." Review of Economics and Statistics, 96.5 (2014): 885897
- Imai, Kosuke, and Marc Ratkovic. "Covariate balancing propensity score." Journal of the Royal Statistical Society: Series B (Statistical Methodology)76.1 (2014): 243-263.

