Instrumental Variables Regression

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Estimating Causal Effects

- A common aim of health services research is the estimation of a causal effect
  - What is the effect of *[treatment]* on *[outcome]*?
- Ideally estimate the effect using a randomized controlled trial
  - Conducting a randomized controlled trial is often not possible
- An alternative is to perform regression analysis using observational data
  - Treatment must be *exogenous*
  - If treatment is not exogenous, estimated effects will be biased
- When treatment is not exogenous, another method is necessary
  - One possibility: instrumental variables (IV) regression

Poll: Familiarity with IV Regression

- New to IV regression
- Somewhat familiar with IV regression
- Advanced knowledge of IV regression

Objectives

- Provide an introduction to instrumental variables (IV) regression
  - Basic linear regression model
  - Necessary conditions for a valid instrument
  - Why and how instrumental variables regression works
  - Examples
  - Limitations

Linear Regression Model

$$Y_i = \beta_0 + \beta_1 X_i + e_i$$

- Y: outcome variable of interest
- X: explanatory variable of interest
- e: error term
  - e contains all other factors besides X that determine the value of Y
- β_1: the change in Y associated with a unit change in X

- In order for $\hat{\beta}_1$ to be an unbiased estimate of the *causal effect* of X on Y, X must be **exogenous**

Exogeneity

- Assumption: $E(e_i|X_i) = 0$
  - Conditional mean of e_i given X_i is zero
  - Additional information in e_i does not help us better predict Y_i
  - X is "exogenous"
  - Implies that X_i and e_i **cannot** be correlated
- X_i and e_i are correlated when there is:
  - Omitted variable bias
  - Sample selection
  - Simultaneous causality
- If X_i and e_i are correlated then X is endogenous
  - $\hat{\beta}_1$ is biased

Intuition

- Idea behind instrumental variables regression:
  - Variation in X has two components
    - One component is correlated with e
      - Causes endogeneity
    - Other component is uncorrelated with e
      - "Exogenous" variation
  - Use only exogenous variation in X to estimate β_1

Instrumental Variables

- Instrumental variables (instruments) can be used to isolate the exogenous variation in X that is uncorrelated with e
- Two conditions for a **valid** instrument
  - Instrument relevance
  - Instrument exogeneity

Regression Model

$$Y_i = \beta_0 + \beta_1 X_i + e_i$$

- Problem: X is endogenous
  - X and e are correlated
- e contains all other factors besides X that determine the value of Y
- Potential instrument Z

Instrument Relevance

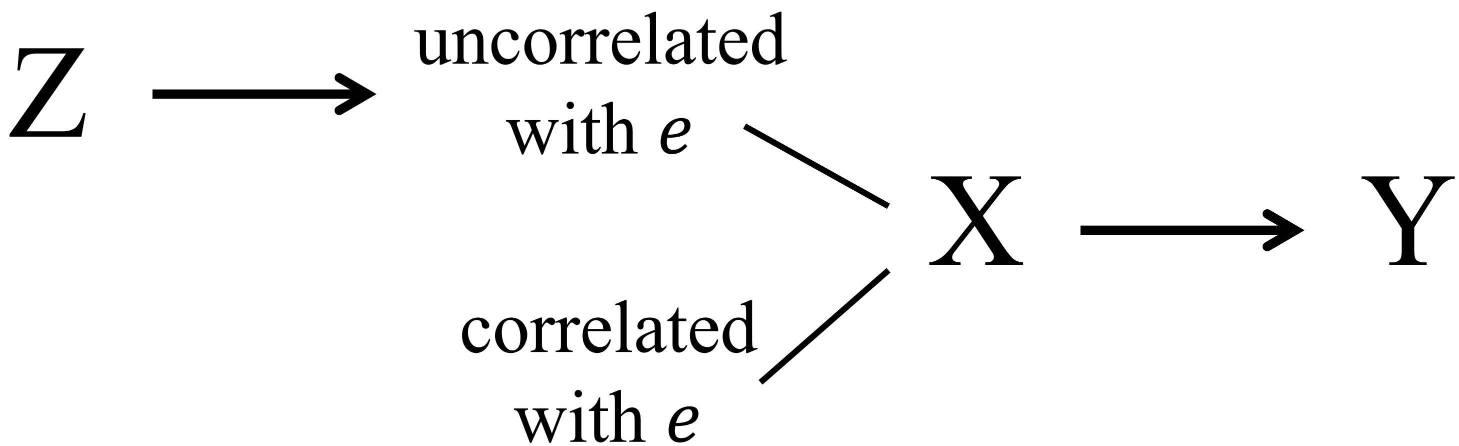
- Instrument relevance: $corr(Z_i, X_i) \neq 0$
  - Z_i is correlated with X_i
  - Variation in Z explains variation in X
  - Z affects X
- Z is "relevant"

Instrument Exogeneity

- Instrument exogeneity: $corr(Z_i, e_i) = 0$
  - Z_i is uncorrelated with e_i
  - Z is uncorrelated with all other factors, besides X, that determine Y
  - Z does **not** affect Y, except through X
- Z is "exogenous"

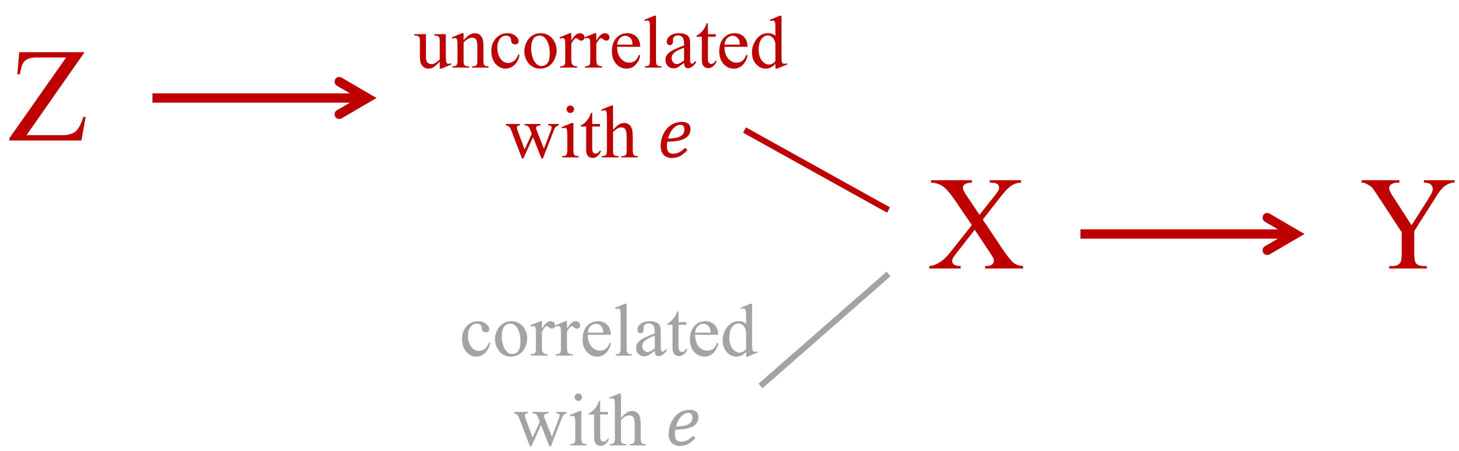
Valid Instrument

$$Y_i = \beta_0 + \beta_1 X_i + e_i$$



Valid Instrument

$$Y_i = \beta_0 + \beta_1 X_i + e_i$$



- *Z* only captures the variation in *X* that is uncorrelated with *e*

Intuition

$$outcome_i = \beta_0 + \beta_1 treatment_i + e_i$$

- Say treatment is assigned through a coin flip:
  - Heads: patient gets treatment
  - Tails: patient does not get treatment
- Is the coin flip a valid instrument for treatment?
  - Does it affect whether or not a patient receives treatment? It is **relevant**.
  - Does it directly affect the outcome? It is **exogenous.**
- Variation in an instrument mimics a randomization of patients to different likelihoods of receiving treatment

Instrumental Variables Model

$$Y_i = \beta_0 + \beta_1 X_i + e_i$$

- Endogeneity: $corr(X_i, e_i) \neq 0$
- Valid instrument, Z:
  - Relevant: $corr(Z_i, X_i) \neq 0$
  - Exogenous: $corr(Z_i, e_i) = 0$

Two Stage Least Squares (1)

- First stage:
  - Regress X on Z:

$$X_i = \underbrace{\pi_0 + \pi_1 Z_i}_{\substack{\text{uncorrelated} \\ \text{with } e}} + \underbrace{\gamma_i}_{\substack{\text{correlated} \\ \text{with } e}}$$

  - Predict X:

$$\hat{X}_i = \hat{\pi}_0 + \hat{\pi}_1 Z_i$$

Two Stage Least Squares (2)

- Second stage:
  - Regress Y on $\hat{X}$:

    $$Y_i = \beta_0{}^{TSLS} + \beta_1{}^{TSLS}\hat{X}_i + error_i$$

  - Estimate $\hat{\beta}_1{}^{TSLS}$
    - $\hat{X}$ is uncorrelated with e from the original regression model $Y_i = \beta_0 + \beta_1 X_i + e_i$
    - $\hat{\beta}_1{}^{TSLS}$ is an unbiased estimate of β_1
    - Note: standard errors in the second stage TSLS regression need to be adjusted

General IV Model

$$Y_i = \beta_0 + \beta_1 X_{1i} + \cdots + \beta_k X_{ki}$$
$$+\beta_{k+1} W_{1i} + \cdots + \beta_{k+r} W_{ri} + e_i$$

- k endogenous regressors: $X_{1i}, \ldots, X_{ki}$
- r exogenous regressors or control variables: $W_{1i}, \ldots, W_{ri}$
- m instrumental variables: $Z_{1i}, \ldots, Z_{mi}$

- There must be at least as many instruments as there are endogenous variables: $m \geq k$

LATE

- IV regression estimates the **local average treatment effect (LATE)**
  - Local average treatment effect: the weighted average of individual causal effects
    - Individuals who are influenced most by the instrument receive the most weight
      - Marginal treatment effect
  - In general, the local average treatment effect differs from the average treatment effect

Intensive Treatment for AMI

- Does more intensive treatment of acute myocardial infarction (AMI) in the elderly reduce mortality?
  - McClellan, McNeil, Newhouse (1994)
- We want to estimate the effect of intensive treatment of AMI (cardiac catheterization, angioplasty, CABG) on mortality

Regression Model

- Model:

$$mortality_i = \beta_0 + \beta_1 treatment_i + e_i$$

Table 2.—Estimated Cumulative Effect of Catheterization, Not Accounting for Selection Bias

Adjustment for Observable Differences Using ANOVA*	Percentage-Point Changes in Mortality Rates (SE)					
	1 d	7 d	30 d	1 y	2 y	4 y
None (unadjusted differences)	−9.4 (0.2)	−18.7 (0.2)	−19.2 (0.3)	−30.5 (0.3)	−34.0 (0.3)	−36.8 (0.3)

- Problem:
  - Whether or not a patient receives more intensive treatment is correlated with many unobserved factors that may also affect mortality
    - E.g., health status, patient or physician preferences

Endogeneity

Table 1.—Characteristics of Elderly Patients With Acute Myocardial Infarction in 1987*

Characteristic	All Patients (N=205 021)	No Catheterization Within 90 d (n=158 261)	Catheterization Within 90 d (n=46 760)
Demographic Characteristics			
Female	50.4	53.5	39.7
Black	5.6	6.0	4.3
Mean age, y (SD)	76.1 (7.2)	77.4 (7.3)	71.6 (5.0)
Urban	70.5	69.6	73.8
Comorbid Disease Characteristics			
Cancer	1.9	2.2	0.8
Pulmonary disease, uncomplicated	10.7	11.1	9.3
Dementia	1.0	1.2	0.1
Diabetes	18.0	18.3	17.1
Renal disease, uncomplicated	1.9	2.3	0.7
Cerebrovascular disease	4.8	5.4	2.8

Endogeneity (2)

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After adjustment for demographic and comorbidity differences	−6.8 (0.2)	−13.5 (0.2)	−17.9 (0.3)	−24.1 (0.3)	−26.6 (0.3)	−28.1 (0.3)

- Evidence of selection bias
  - Estimates that do not account for selection are biased

Instrument

- Idea:
  - Patients who live closer to hospitals that have the capacity to perform more intensive treatments are more likely to receive those treatments (relevance)
  - The distance a patient lives from a given hospital should be independent of his health status (exogeneity)
- Instrument (for intensive treatment): differential distance to catheterization and revascularization hospitals

Instrument (2)

Table 4.—Patient Characteristics by Differential Distance to a Catheterization or Revascularization Hospital*

Characteristic	Differential Distance ≤2.5 Miles (n=102 516)	Differential Distance >2.5 Miles (n=102 505)
Comorbid Disease Characteristics		
Cancer	1.9	1.9
Pulmonary disease, uncomplicated	10.4	10.9
Dementia	0.99	0.94
Diabetes	18.1	18.0
Renal disease, uncomplicated	2.0	1.9
Cerebrovascular disease	4.8	4.8
Treatments		
Initial admit to catheterization hospital†	34.4	5.0
Initial admit to revascularization hospital†	41.7	10.7
Catheterization within 7 d	20.7	11.0
Catheterization within 90 d	26.2	19.5
CABG‡ within 90 d	8.6	6.9
PTCA§ within 90 d	6.4	4.3

Results

Table 7.—Instrumental Variable Estimates of the Effects of Patient Location, High-Volume Hospital, and Catheterization on Mortality at Indicated Time Intervals After Acute Myocardial Infarction

Average Effect	Time After Acute Myocardial Infarction, Percentage-Point Change (SE)						
	1 d	7 d	30 d	1 y	2 y	3 y	4 y
Catheterization within 90 d							
Cumulative	−8.8 (2.0)	−11.5 (2.5)	−7.4 (2.9)	−4.8 (3.2)	−5.4 (3.3)	−5.0 (3.2)	−5.1 (3.2)

Table 2.—Estimated Cumulative Effect of Catheterization, Not Accounting for Selection Bias

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- IV estimates of the effect of catheterization on mortality are much smaller than estimates that do not take into account selection

Results (2)

Table 7.—Instrumental Variable Estimates of the Effects of Patient Location, High-Volume Hospital, and Catheterization on Mortality at Indicated Time Intervals After Acute Myocardial Infarction

	Time After Acute Myocardial Infarction, Percentage-Point Change (SE)						
Average Effect	**1 d**	**7 d**	**30 d**	**1 y**	**2 y**	**3 y**	**4 y**
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- Catheterization within 90 days of AMI reduces mortality by 5 percentage points at 1 to 4 years
- Caveats:
  - The validity of results hinge on the validity of the instrument
  - IV estimates the LATE: this is an estimate of the *marginal effect* of catheterization (for patients who would not have otherwise received treatment if they lived differentially far from a catheterization or revascularization hospital)
  - This estimate is an upper bound of the effect of catheterization
    - If catheterization or revascularization hospitals offer better care other than more intensive procedures (e.g., more beds, specialists, ICU), then mortality should be lower at those hospitals

Distance as an Instrument?

- What is the effect of primary care (PC) on health outcomes?
  - Endogeneity: people usually see a doctor when they are sick
  - Suppose:
    - Patients who live closer to PC clinics are more likely to see a PC provider
    - Patients who need to see a doctor often move to live closer to clinics
  - Can we use distance to the nearest PC clinic as an instrument for PC use?

Polls: Distance as an Instrument?

1. Is distance relevant?
   A. Yes
   B. No
2. Is distance exogenous?
   A. Yes
   B. No

Distance as an Instrument? (2)

- What is the effect of emergency department (ED) services for car accident injuries on mortality?
  - Endogeneity: only seriously injured passengers are taken to the ED
  - Suppose:
    - All people who need medical care are taken to the ED, regardless of distance
    - Distance to the nearest ED is probably uncorrelated with accident severity
  - Can we use distance to the nearest ED as an instrument for treatment in an ED?

Polls: Distance as an Instrument?

1. Is distance relevant?
   A. Yes
   B. No
2. Is distance exogenous?
   A. Yes
   B. No

Other IV Examples

- Zulman, Pal Chee, et al. (2015): effect of VA intensive management primary care on VA health care costs; instrument: random assignment to treatment vs. usual care groups
- Bhattacharya, et al. (2011): effect of insurance coverage on body weight; instruments: distribution of firm size and Medicaid coverage for each state and year
- Doyle (2013): effect of foster care on long- and short-term outcomes; instrument: random assignment to investigators

Weak Instruments

- Instruments that explain little variation in X are **weak**
- IV regression with weak instruments provide unreliable estimates
- Rule of thumb to check for weak instruments when there is only one endogenous regressor:
  - From the first stage regression of TSLS, compute the F-statistic testing the hypothesis that the coefficients on the instruments are all equal to zero

$$X_i = \pi_0 + \pi_1 Z_{1i} + \cdots + \pi_m Z_{mi} + \gamma_i$$

$$H_0 \colon \pi_1 = \ldots = \pi_m = 0$$
$$H_1 \colon \pi_1 \neq 0 \; or \ldots or \; \pi_m \neq 0$$

  - F-statistic > 10 indicates instruments are not weak
  - Note: this is a rule of thumb; we still need a convincing argument that the instrument is relevant (strong)

Endogenous Instruments

- Instruments that are correlated with the error term (other factors that affect the outcome variable) are **endogenous**
- IV regression with endogenous instruments provide unreliable estimates
  - The point of IV regression is to isolate and utilize exogenous variation in X to estimate β_1
- When there are more instruments than there are endogenous regressors, possible to test "overidentifying restrictions"
  - Overidentifying restrictions test (J-statistic)
- Need a convincing argument that the instruments are exogenous

Summary

- IV regression is powerful tool to estimate causal effects
- Conditions for a valid instrument:
  - Relevance: the instrument must affect treatment
  - Exogeneity: the instrument must be uncorrelated with all other factors that may affect outcomes
- Good instruments are difficult to find
- Using an invalid (weak or endogenous) instrument will give meaningless results
- Some tests available to check instrument validity, but what is absolutely necessary is a good "story" for why an instrument is relevant and exogenous

Resources and References

- Stock, James H. and Mark W. Watson, 2011. Introduction to Econometrics (Third Edition). Boston, MA: Addison-Wesley. (Chapter 12: Instrumental Variables Regression)
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