

Limited Dependent Variables

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Limited Dependent Variables

- 0-1, small number of options, small counts, etc.
 - The dependent variable is not continuous, or even close to continuous.
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Outline

- Binary Choice
 - Multinomial Choice
 - Counts
 - Most models in general framework of probability models
 - Prob (event/occurs)
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Basic Problems

- Heteroscedastic error terms
 - Predictions not constrained to match actual outcomes
 - Real problem with predicted values being negative when a negative number isn't possible
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$$Y_i = \beta_0 + \beta X + \varepsilon_i$$

$Y_i=0$ if lived, $Y_i=1$ if died

$$\text{Prob}(Y_i=1) = F(X, \beta)$$

$$\text{Prob}(Y_i=0) = 1 - F(X, \beta)$$

OLS, also called a linear probability model

ε_i is heteroscedastic, depends on βX

Predictions not constrained to $(0,1)$

Binary Outcomes Common in Health Care

- Mortality
 - Other outcome
 - Infection
 - Patient safety event
 - Rehospitalization <30 days
 - Decision to seek medical care
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Standard Approaches to Binary Choice-1

- Logistic regression

$$\textit{prob}(Y=1) = \frac{e^{\beta X}}{1 + e^{\beta X}}$$

Advantages of Logistic Regression

- Designed for relatively rare events
 - Commonly used in health care; most readers can interpret an odds ratio
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Standard Approaches to Binary Choice-2

- Probit regression (classic example is decision to make a large purchase)

$$y^* = \beta X + \varepsilon$$

$$y=1 \text{ if } y^* > 0$$

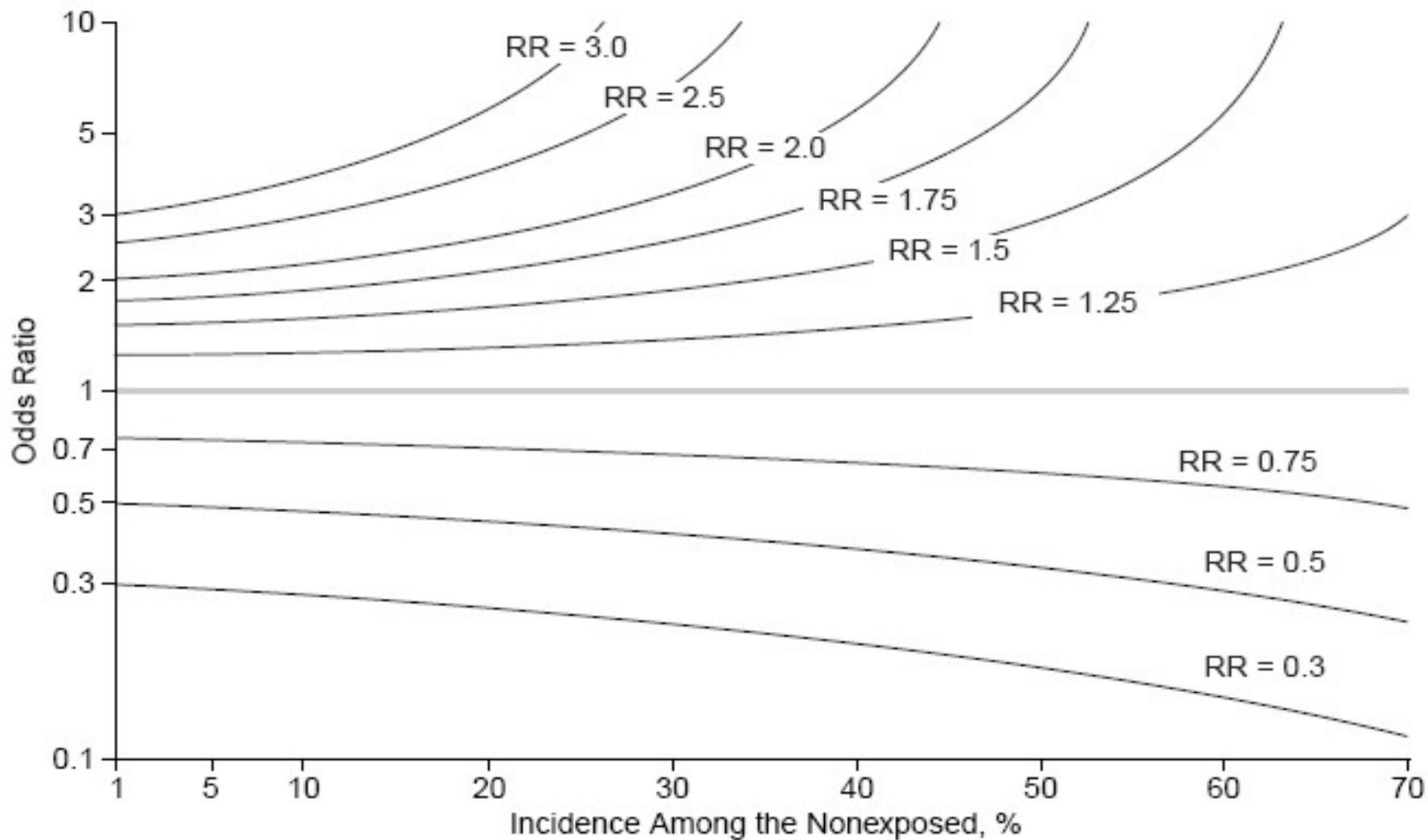
$$y=0 \text{ if } y^* \leq 0$$

Binary Choice

- There are other methods, using other distributions.
 - In general, logistic and probit give about the same answer.
 - It used to be a lot easier to calculate marginal effects with probit, not so any more
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Odds Ratios vs. Relative Risks

- Standard method of interpreting logistic regression is odds ratios.
 - Convert to % effect, really relative risk
 - This approximation starts to break down at 10% outcome incidence
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The relationship between risk ratio (RR) and odds ratio by incidence of the outcome.

Can Convert OR to RR

- Zhang J, Yu KF. What's the Relative Risk? A Method of Correcting the Odds Ratio in Cohort Studies of Common Outcomes. JAMA 1998;280(19):1690-1691.

$$\mathbf{RR = \frac{OR}{(1-P_0) + (P_0 \times OR)}}$$

Where P_0 is the sample probability of the outcome

Effect of Correction for RR

From Phibbs et al., NEJM 5/24/2007, $\approx 20\%$ mortality

Odds Ratio	Calculated RR
2.72	2.08
2.39	1.91
1.78	1.56
1.51	1.38
1.08	1.06

Extensions

- Panel data, can now estimate both random effects and fixed effects models. The Stata manual lists over 30 related estimation commands
 - All kinds of variations.
 - Panel data
 - Grouped data
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Extensions

- Goodness of fit tests. Several tests.
 - Probably the most commonly reported statistics are:
 - Area under ROC curve, c-statistic in SAS output. Range 0.50 to 1.0.
 - Hosmer-Lemeshow test
 - NEJM paper, $c=0.86$, H-L $p=0.34$
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More on Hosmer-Lemeshow Test

- The H-L test breaks the sample up into n (usually 10, some programs (Stata) let you vary this) equal groups and compares the number of observed and expected events in each group.
 - If your model predicts well, the events will be concentrated in the highest risk groups; most can be in the highest risk group.
 - Alternate specification, divide the sample so that the events are split into equal groups.
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Estimate Note for Very Large Samples

- If you have very large samples; millions, it takes a lot longer to estimate a maximum likelihood model than OLS
 - But, same X matrix, so the p-values for OLS are approximately the same as a logit model. Can use OLS for model development, and only estimate the final models with logit or other maximum likelihood model.
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Multinomial Choice

- What if more than one choice or outcome?
 - Options are more limited
 - Multivariable Probit (multiple decisions, each with two alternatives)
 - Two different logit models (single decision, multiple alternatives)
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Logit Models for Multiple Choices

- Conditional Logit Model (McFadden)
 - Unordered choices
 - Multinomial Logit Model
 - Choices can be ordered.
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Examples of Health Care Uses for Logit Models for Multiple Choices

- Choice of what hospital to use, among those in market area
 - Choice of treatment among several options
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Conditional Logit Model

$$\text{prob}(Y_i = j) = \frac{e^{\beta X_{ij}}}{\sum_{j=1}^J e^{\beta X_{ij}}}$$

Conditional logit model

- Also known as the random utility model
 - Is derived from consumer theory
 - How consumers choose from a set of options
 - Model driven by the characteristics of the choices.
 - Individual characteristics “cancel out” but can be included indirectly. For example, in hospital choice, can interact individual characteristic with distance to hospital
 - Can express the results as odds ratios.
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Estimation of McFadden's Model

- Some software packages (e.g. SAS) require that the number of choices be equal across all observations.
 - LIMDEP, allows a “NCHOICES” options that lets you set the number of choices for each observation. This is a very useful feature. May be able to do this in Stata (clogit) with “group”
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Example of Conditional Logit Estimates

- Study I did looking at elderly service-connected veterans choice of VA or non-VA hospital

Log distance	0.66	$p < 0.001$
VA	2.80	$p < 0.001$

Multinomial Logit Model

$$\text{prob}(Y = j) = \frac{e^{\beta_j X_i}}{1 + \sum_{j=1}^J e^{\beta_j X_i}}$$

$$\text{prob}(Y = 0) = \frac{1}{1 + \sum_{j=1}^J e^{\beta_j X_i}}$$

Multinomial Logit Model

- Must identify a reference choice, model yields set of parameter estimates for each of the other choices, relative to the reference choice
 - Allows direct estimation of parameters for individual characteristics. Model can (should) also include parameters for choice characteristics
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Independence of Irrelevant Alternatives

- Results should be robust to varying the number of alternative choices
 - Can re-estimate model after deleting some of the choices.
 - McFadden, regression based test. Regression-Based Specification Tests for the Multinomial Logit Model. *J Econometrics* 1987;34(1/2):63-82.
 - If fail IIA, may need to estimate a nested logit model
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Independence of Irrelevant Alternatives - 2

- McFadden test can also be used to test for omitted variables.
 - For many health applications, doesn't matter, the models are very robust (e.g. hospital choice models driven by distance).
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Count Data (integers)

- Continuation of the same problem; dependent variable can only assume specific values and can't be $< \text{zero}$
 - Problem diminishes as counts increase
 - Rule of Thumb. Need to use count data models for counts under 30
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Count Data

- Some examples of where count data models are needed in health care
 - Dependent variable is number of outpatient visits
 - Number of times a prescription of a chronic disease medication is refilled in a year
 - Number of adverse events in a unit (or hospital) over a period of time
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Count Data

- Poisson distribution. A distribution for counts.
 - Problem: very restrictive assumption that mean and variance are equal

$$\text{prob}(Y_i = y_i) = \frac{e^{-\lambda_i} \lambda_i^{y_i}}{y_i!}$$

Count Data

- In general, negative binomial is a better choice. Stata (nbreg), test for what distribution is part of the package. Other distributions can also be used.

$$f(y_i | x_i, u_i) = \frac{e^{-\lambda u_i} (\lambda u_i)^{y_i}}{y_i!}$$

Interpreting Count Data Models

- $\ln E(\text{event rate}) = Bx$
 - Incidence Rate Ratio = e^B , like on odds ratio, with a similar interpretation
 - Example, effect of average RN tenure on unit on infection rate
 - $B = -0.262$ $IRR = 0.770$
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Notes for Count Data Models

- More common to see OLS used for counts than for binary or very limited choices.
 - Real problem with OLS when there are lots of zeros. Will result in reduced statistical significance. Can go in opposite direction when counts are larger.
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Example, OLS vs. Count Data Model, Effect on Statistical Significance (t-values)

	Count Data Model	OLS
RN HPPD	0.18	-1.44
LPN HPPD	-1.78	-3.62
Aide HPPD	0.65	-1.92

Notes for Count Data Models-2

- 30 is a rule of thumb, but should still consider a count model if most are small counts
 - Need to consider distribution and data generating process. If mixed process, may need to split sample
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Example of Mixed Data Generating Processes

- Predicting LOS for newborns
 - Well babies, all with $LOS \leq 5$ days, clearly a count
 - Sick newborns, can have very long LOS
 - Solution, separate samples, use count model for well babies and OLS for sick babies
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Other Models

- New models are being introduced all of the time. More and better ways to address the problems of limited dependent variables.
 - Includes semi-parametric and non-parameteric methods.
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Reference Texts

- Greene. Econometric Analysis
 - Wooldridge. Econometric Analysis of Cross Section and Panel Data
 - Maddala. Limited-Dependent and Qualitative Variables in Econometrics
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Journal References

- McFadden D. Specification Tests for the Multinomial Logit Model. *J Econometrics* 1987;34(1/2):63-82.
 - Zhang J, Yu KF. What's the Relative Risk? A Method of Correcting the Odds Ratio in Cohort Studies of Common Outcomes. *JAMA* 1998;280(19):1690-1691.
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Next lecture

Cost as the Dependent Variable

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