Econometrics Course: Cost as the Dependent Variable (I)

Paul G. Barnett, PhD

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Health Economics Resource Center

HSR&D HEALTH SERVICES RESEARCH & DEVELOPMENT

What is health care cost?

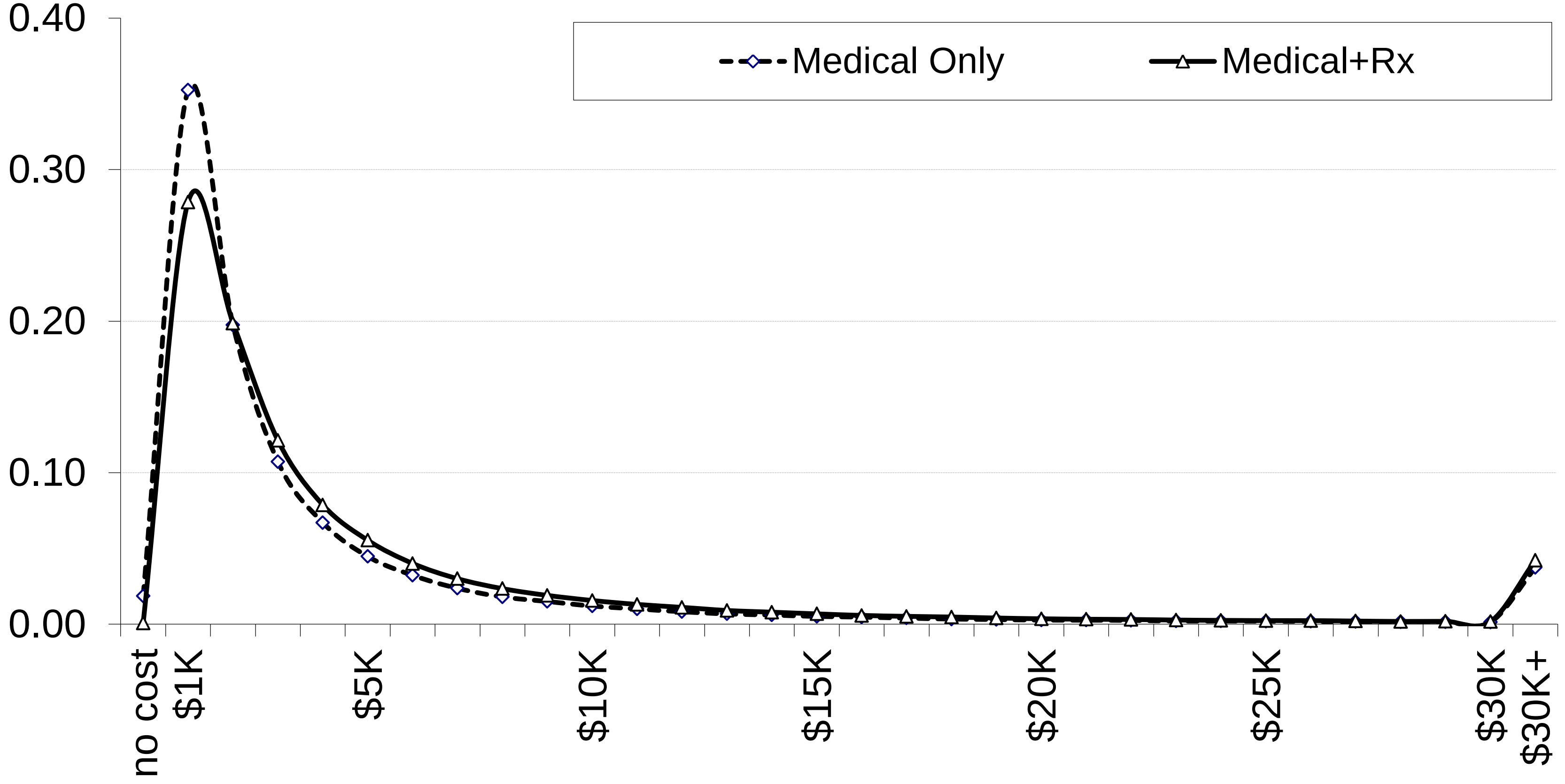
- Cost of an intermediate product, e.g.,
  - chest x-ray
  - a day of stay
  - minute in the operating room
  - a dispensed prescription
- Cost of a bundle of products
  - Outpatient visit
  - Hospital stay

What is health care cost (cont.)?

- Cost of a treatment episode
  - visits and stays over a time period
- Annual cost
  - All care received in the year

Annual per person VHA costs FY10

(5% random sample)



Descriptive statistics: VHA costs FY10

(5% sample, includes outpatient pharmacy)

	Cost
Mean	5,768
Median	1,750
Standard Deviation	18,874
Skewness	13.98
Kurtosis	336.3

Skewness and kurtosis

- Skewness (3^{rd} moment)
  - Degree of symmetry
  - Skewness of normal distribution =0
  - Positive skew: more observations in right tail
- Kurtosis (4^{th} moment)
  - Peakness of distribution and thickness of tails
  - Kurtosis of normal distribution=3

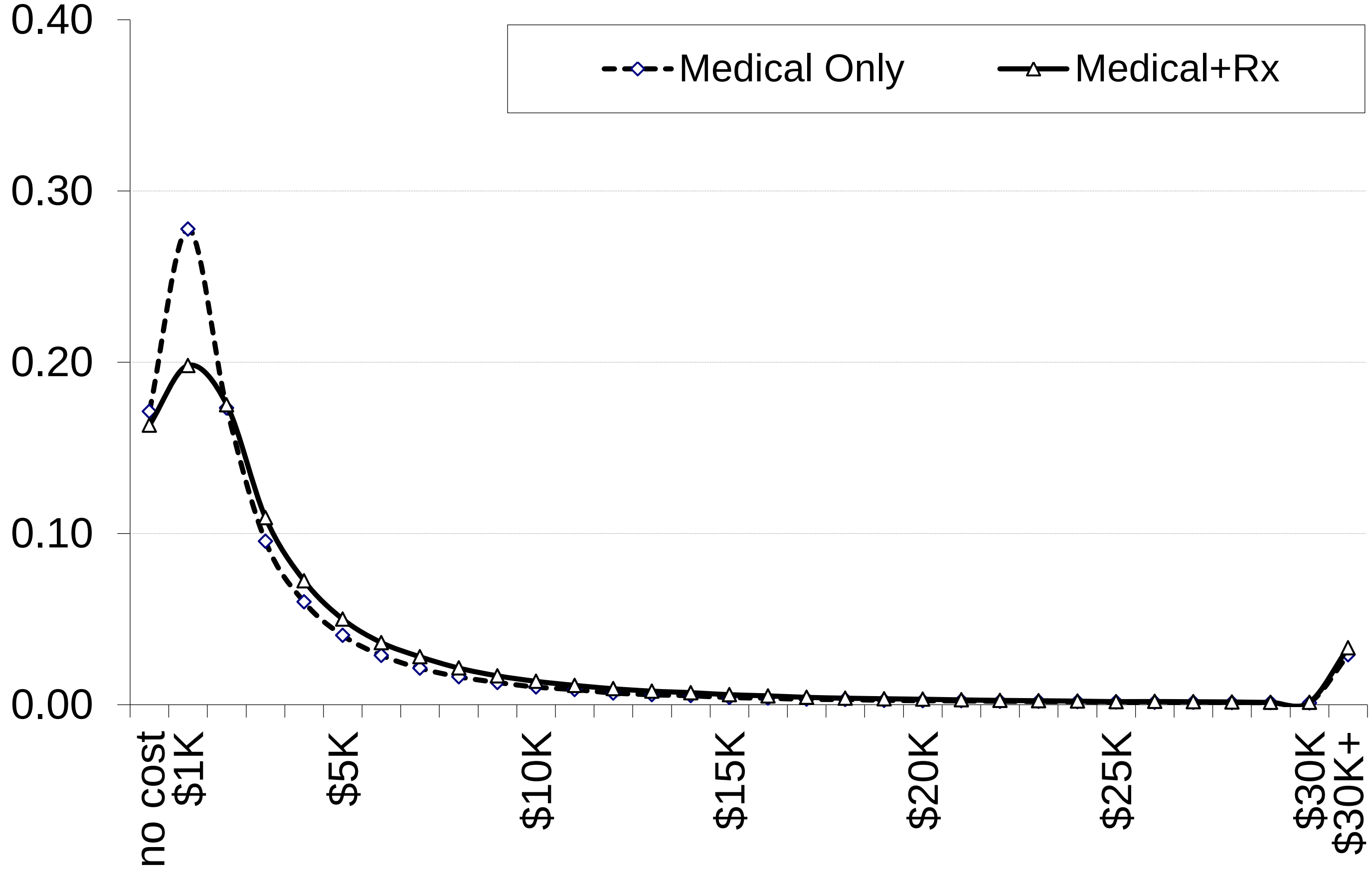
Distribution of cost: skewness

- Rare but extremely high cost events
  - E.g. only some individuals hospitalized
  - Some individuals with expensive chronic illness
- Positive skewness (skewed to the right)

Comparing the cost incurred by members of two groups

- Do we care about the mean or the median?

Annual per person VHA costs FY10 *among those who used VHA in FY09*



Distribution of cost: zero value records

- Enrollees who don’t use care
  - Zero values
  - Truncation of the distribution

What hypotheses involving cost do you want to test?

What hypotheses involving cost do you want to test?

- I would like to learn how cost is affected by:
  – Type of treatment
  – Quantity of treatment
  – Characteristics of patient
  – Characteristics of provider
  – Other

Review of Ordinarily Least Squares (OLS)

- Also known as: Classic linear model
- We assume the dependent variable can be expressed as a linear function of the chosen independent variables, e.g.:
- $Y_i = \alpha + \beta X_i + \varepsilon_i$

Ordinarily Least Squares (OLS)

- Estimates parameters (coefficients) α, β
- Minimizes the sum of squared errors
  - (the distance between data points and the regression line)

Linear model

- Regression with cost as a linear dependent variable (Y)
  - $Y_i = \alpha + \beta X_i + \varepsilon_i$
- β is interpretable in raw dollars
  - Represents the change of cost (Y) for each unit change in X
  - E.g. if β=10, then cost increases $10 for each unit increase in X

Expected value of a random variable

- E(random variable)
- $E(W) = \Sigma W_i \, p(W_i)$
  - For each i, the value of W_i times probability that W_i occurs
  - Probability is between 0 and 1
  - A weighted average, with weights by probability

Expected value of a random variable (cont)

- Expected value from a roll of the die
- $E(W) = \Sigma\ W_i\ p(W_i)$

$$1\left(\frac{1}{6}\right)+2\left(\frac{1}{6}\right)+3\left(\frac{1}{6}\right)+4\left(\frac{1}{6}\right)+5\left(\frac{1}{6}\right)+6\left(\frac{1}{6}\right)$$

$$=3.5$$

Review of OLS assumptions

- Expected value of error is zero $E(\varepsilon_i)=0$
- Errors are independent $E(\varepsilon_i \varepsilon_j)=0$
- Errors have identical variance $E(\varepsilon_i^2)=\sigma^2$
- Errors are normally distributed
- Errors are not correlated with independent variables $E(X_i \varepsilon_i)=0$

When cost is the dependent variable

- Which of the assumptions of the classical model are likely to be violated by cost data?
  - Expected error is zero
  - Errors are independent
  - Errors have identical variance
  - Errors are normally distributed
  - Error are not correlated with independent variables

Compare costs incurred by members of two groups

- Regression with one dichotomous explanatory variable
- $Y = \alpha + \beta X + \varepsilon$
- Y cost
- X group membership
  - 1 if experimental group
  - 0 if control group

Predicted difference in cost of care for two group

$$Y = \alpha + \beta X + \varepsilon$$

Predicted value of Y conditional on X=0

(Estimated mean cost of control group)

$$\hat{Y} \mid (X = 0) = \alpha$$

- Predicted Y when X=1

  (Estimated mean cost experimental group)

$$\hat{Y} \mid (X = 1) = \alpha + \beta$$

Other statistical tests are special cases

- Analysis of Variance (ANOVA) is a regression with one dichotomous independent variable
- Relies on OLS assumptions

Compare groups controlling for case mix

- Include case-mix variable, Z

$$Y = \alpha + \beta_1 X + \beta_2 Z + \varepsilon$$

Compare groups controlling for case mix (cont).

- Estimated mean cost of control group controlling for case mix (evaluated at mean value for case-mix variable)

$$\hat{Y} \mid (X = 0) = \alpha + \beta_2 \bar{Z}$$

$$where\ \bar{Z}\ is\ mean\ of\ \ Z$$

Compare groups controlling for case mix (cont).

- Estimated mean cost of experimental group controlling for case mix (evaluated at mean value for case-mix variable)

$$\hat{Y} \mid (X = 1) = \alpha + \beta_1 + \beta_2 \overline{Z}$$

$$\textit{where } \overline{Z} \textit{ is mean of } Z$$

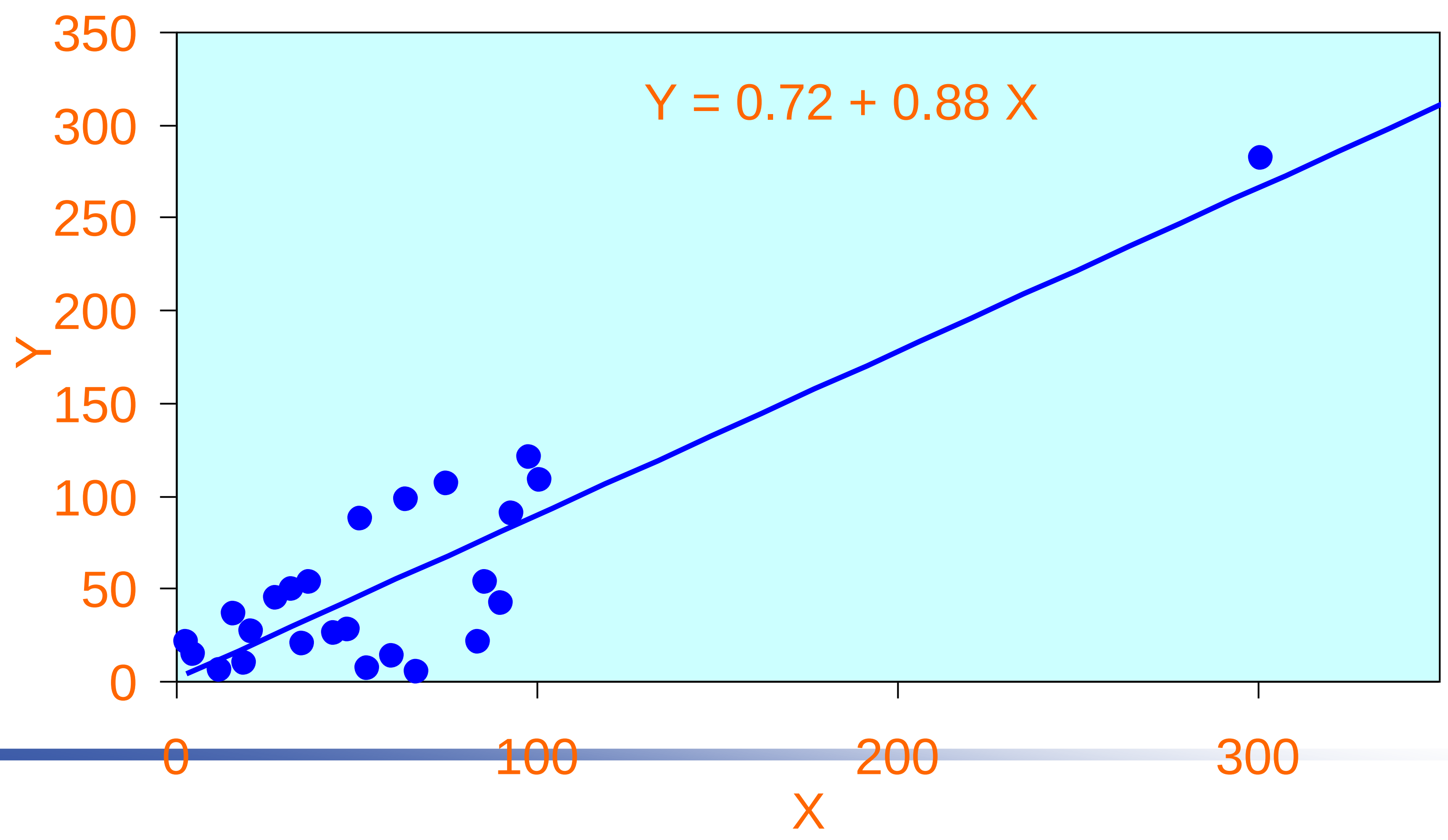
Assumptions are about error term

- Formally, the OLS assumptions are about the error term
- The residuals (estimated errors) often have a similar distribution to the dependent variable

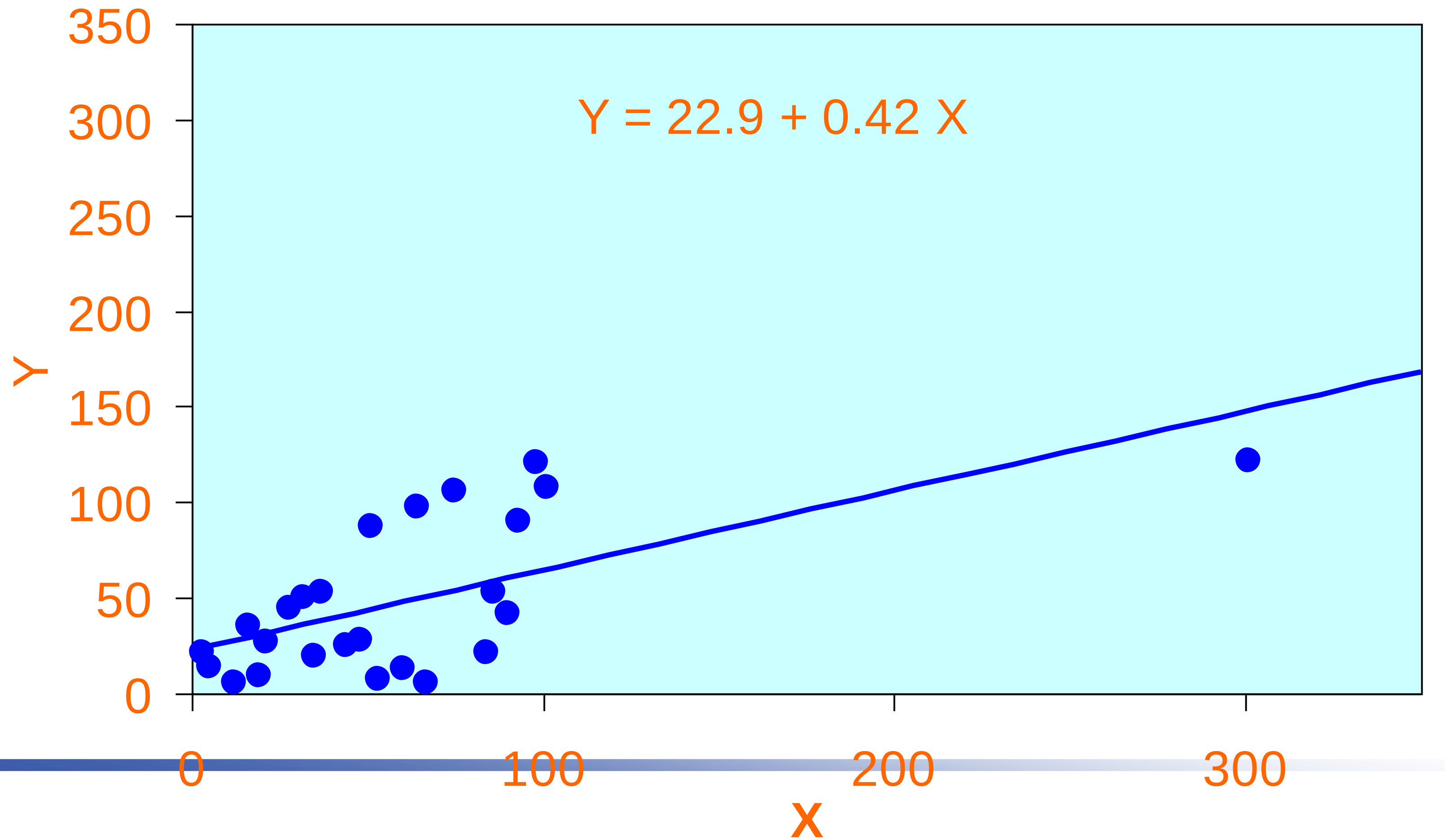
Why worry about using OLS with skewed (non-normal) data?

- "In small and moderate sized samples, a single case can have tremendous influence on an estimate"
  - – Will Manning
  - – Elgar Companion to Health Economics AM Jones, Ed. (2006) p. 439
- There are no values skewed to left to balance this influence
- In Rand Health Insurance Experiment, one observation accounted for 17% of the cost of a particular health plan

The influence of a single outlier observation



The influence of a single outlier observation

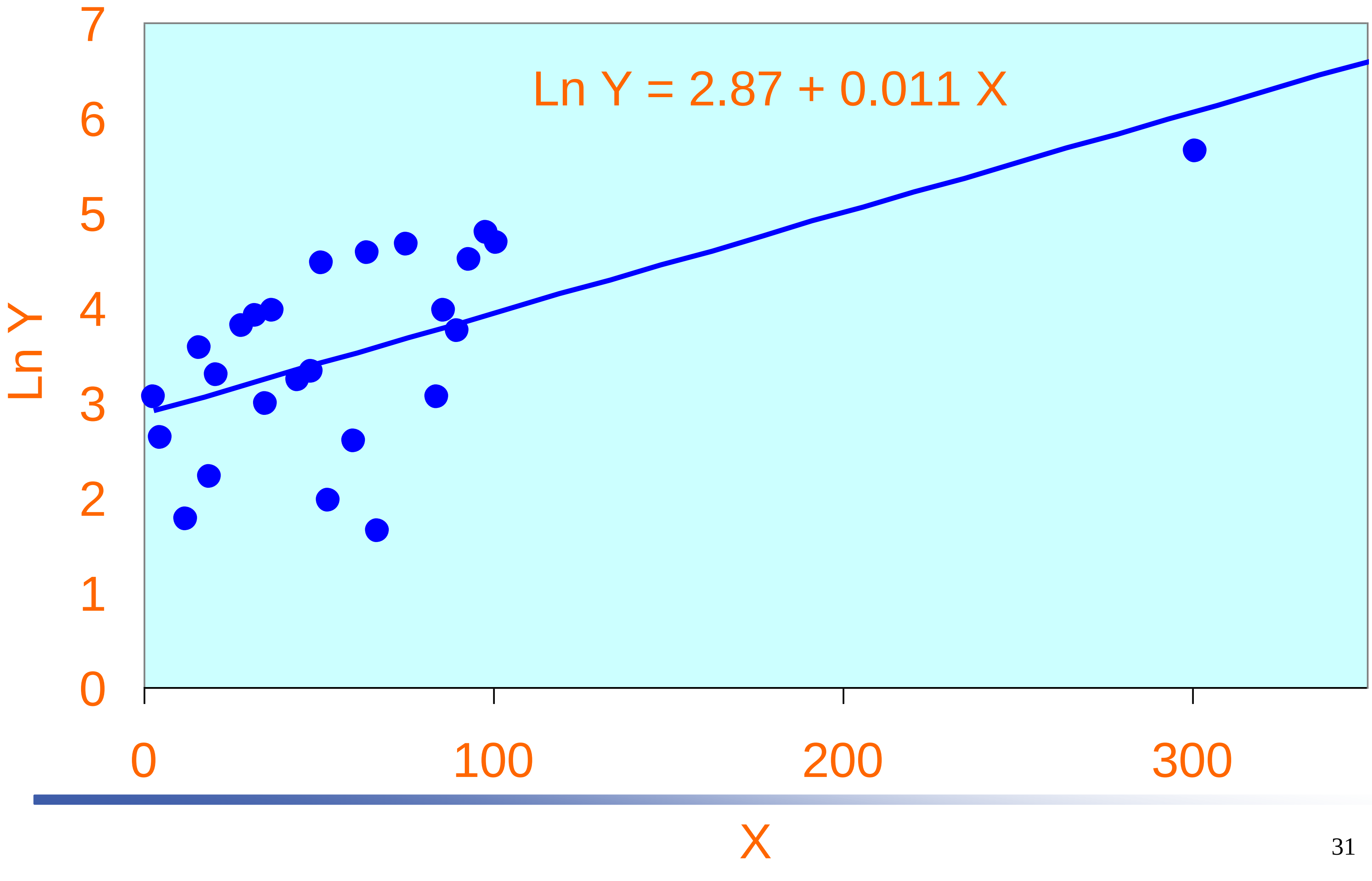


Log Transformation of Cost

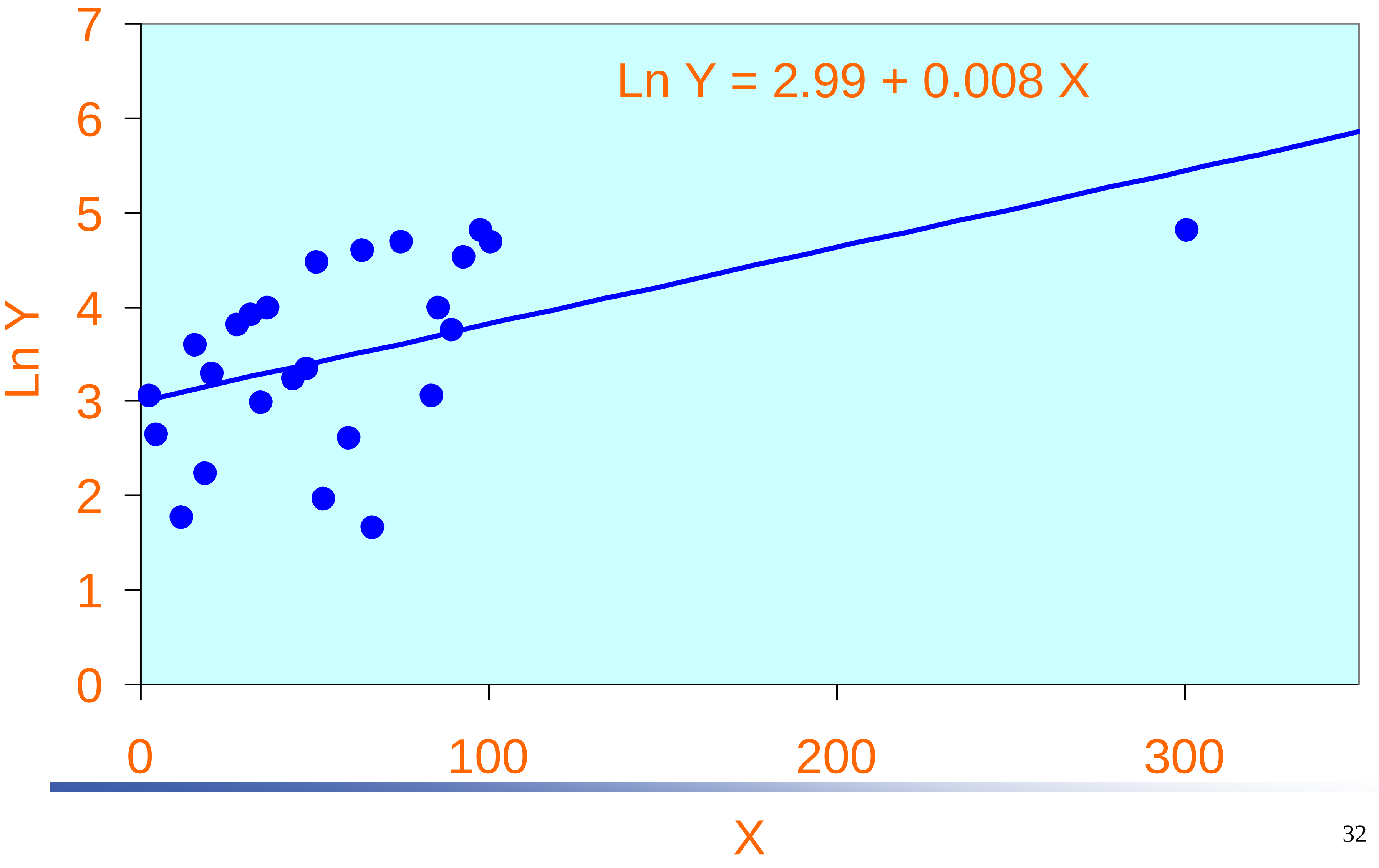
- Take natural log (log with base e) of cost
- Examples of log transformation:

COST	LN(COST)
$10	2.30
$1,000	6.91
$100,000	11.51

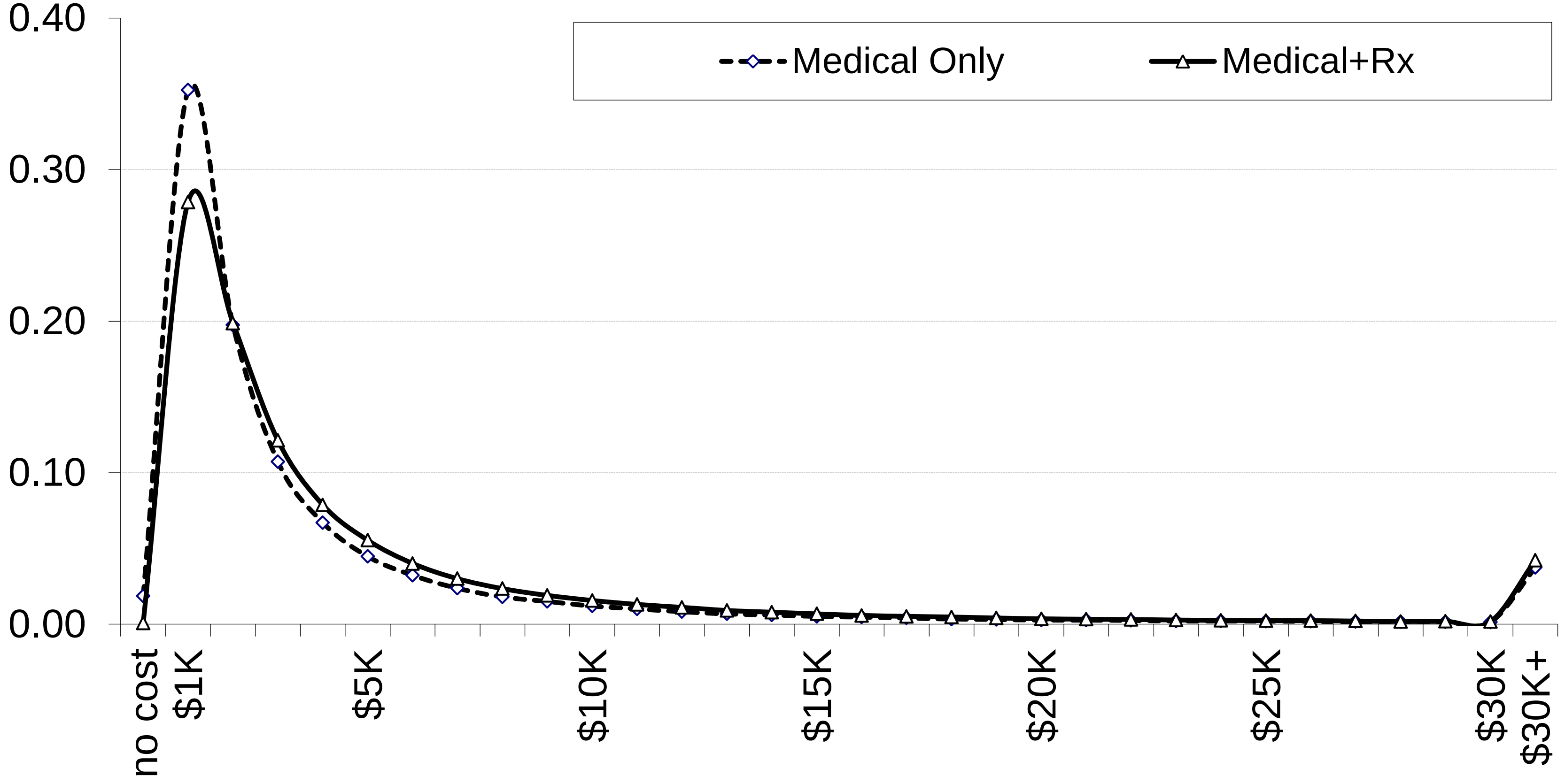
Same data- outlier is less influential



Same data- outlier is less influential

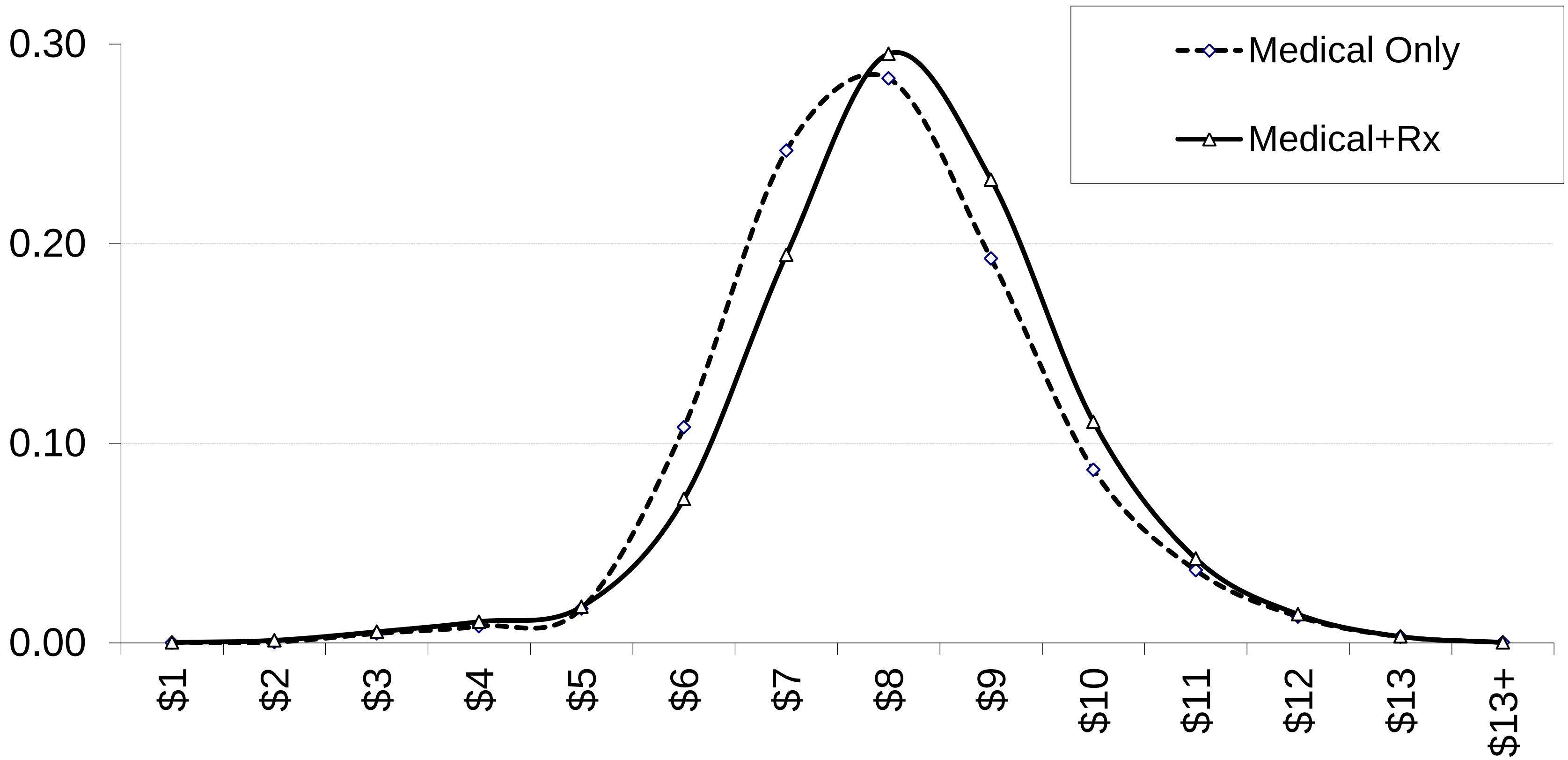


Annual per person VHA costs FY10



Effect of log transformation

Annual per person VHA costs FY10



Descriptive statistics: VHA costs FY10

(5% sample, includes outpatient pharmacy)

	Cost	Ln Cost
Mean	5,768	7.68
Median	1,750	7.67
Standard Deviation	18,874	1.50
Skewness	13.98	-0.18
Kurtosis	336.3	1.12

Log linear model

- Regression with log dependent variable

$$\text{Ln Y} = \alpha + \beta \text{X} + \mu$$

Log linear model

- $\text{Ln}(Y) = \alpha + \beta X + \mu$
- Parameters (coefficients) are not interpretable in raw dollars
  - Parameter represents the relative change of cost (Y) for each unit change in X
  - E.g. if $\beta=0.10$, then cost increases 10% for each unit increase in X

What is the mean cost of the experimental group controlling for case-mix?

- We want to find the fitted value of Y
- Conditional on X=1
- With covariates held at the mean

$$\mathrm{Ln}\,(\mathrm{Y}) = \alpha + \beta_1\,\mathrm{X} + \beta_2\overline{\mathrm{Z}} + \mu$$

$$\textit{What is } \hat{Y}\,?$$

Can we retransform by taking antilog of fitted values?

$$\text{With the model}:$$

$$\mathrm{Ln}\,(\mathrm{Y}) = \alpha + \beta_1\,\mathrm{X} + \beta_2 \mathrm{Z} + \mu$$

Does

$$\hat{Y} = e^{\alpha + \beta_1 \mathrm{X} + \beta_2 Z}\ ?$$

What is fitted value of Y?

$$E(Y) = E(e^{\alpha+\beta_1 X+\beta_2 Z+\mu_i})$$

$$= e^{\alpha+\beta_1 X+\beta_2 Z} E(e^{\mu_i})$$

$$= e^{\alpha+\beta_1 X+\beta_2 Z}$$

$$only\ if\ we\ can\ assume:$$

$$E(e^{\mu_i}) = 1$$

Retransformation bias

$$Since\ E(\mu_i) = 0$$

$$does\, E(e^{\mu_i}) = 1 \quad ?$$

$$Does\ e^{E(\mu_i)} = E(e^{\mu_i})?$$

Retransformation bias

$$Example\ of\ \ why\ E(e^{\mu_i}) \neq e^{E(\mu_i)}$$

$$when\ \mu_1 = 1\, and\ \mu_2 = -1:$$

$$e^{E(\mu^i)} = e^{+1-1} = e^0 = 1$$

$$E(e^{\mu_i}) = \frac{e^1 + e^{-1}}{2} = \frac{2.72 + 0.37}{2} = 1.5$$

Retransformation bias

- The expected value of the antilog of the residuals

  does not equal

- The antilog of the expected value of the residuals

$$E(e^{\mu_i}) \neq e^{E(\mu_i)}!$$

One way to eliminate retransformation bias: the smearing estimator

$$E(Y) = E(e^{\alpha+\beta X_1+\beta Z_2+\mu_i})$$

$$= \left(e^{\alpha+\beta X_1+\beta Z_2}\right)E(e^{\mu_i})$$

$$= \left(e^{\alpha+\beta X_1+\beta Z_2}\right)\frac{1}{n}\sum_{i=1}^{n}(e^{\mu_i})$$

Smearing Estimator

$$\frac{1}{n}\sum_{i=1}^{n}(e^{\mu_i})$$

Smearing estimator

- This is the mean of the anti-log of the residuals
- Most statistical programs allow you to save the residuals from the regression
  - Find their antilog
  - Find the mean of this antilog
- The estimator is often greater than 1

Smearing estimator in SAS

- Save the residuals from the regression
  - PROC REG;
  - MODEL LNCOST=GROUP;
  - OUTPUT OUT=my_file RESIDUALS=my_residuals
- Find their antilog
  - DATA my_file; SET my_file;
  - my_smear=EXP(my_residuals);
- Find the mean
  - PROC MEANS DATA=my_file MEAN;
  - VAR my_smear;

Smearing estimator in SAS

- Save mean of my_smear
  - as &smear_est (a single value)
- Predict the log cost (PREDICTED)
- Transform back into natural units (dollars of cost)

  Predicted_Cost =
  EXP(Predicted_lncost)*&smear_ext;

Predicting the log cost

- Simple approach: use coefficients
  - INTERCEPT (predicted log cost if not in the group)
  - INTERCEPT + BETA (predicted log cost if in the group)
  - This yields two predicted log cost
  - Retransform to dollars cost with smearing correction and compare them
- What if there are other covariates besides GROUP?
  - Evaluate parameters with covariates set to their mean?

The simple approach to comparing groups controlling for case mix

- Estimated mean log cost of control group controlling for case mix (evaluated at mean value for case-mix variable)

$$LN\hat{C}ost \mid (X = 0) = \alpha + \beta_2 \bar{Z} \, ?$$

- This can lead to bias
- The antilog of the mean is ***<u>not</u>*** the mean of the antilog!

Predicting the log cost (continued)

- When there are other variables in the regression
- Predict log cost for each scenario given the values of other variables for that observation
  - "as if" observation was in the group (set GROUP to ONE)
  - "as if" observation wasn't in the group (set GROUP to ZERO)
- Retransform each predicted cost to "natural units of cost" with smearing estimator
- Find mean of the N observations for each scenario
- Compare the means of the corrected predictions

Correcting retransformation bias

- See Duan J Am Stat Assn 78:605
- Smearing estimator assumes identical variance of errors (homoscedasticity)
- Other methods when this assumption can't be made

Retransformation

- Log models can be useful when data are skewed
- Fitted values must correct for retransformation bias

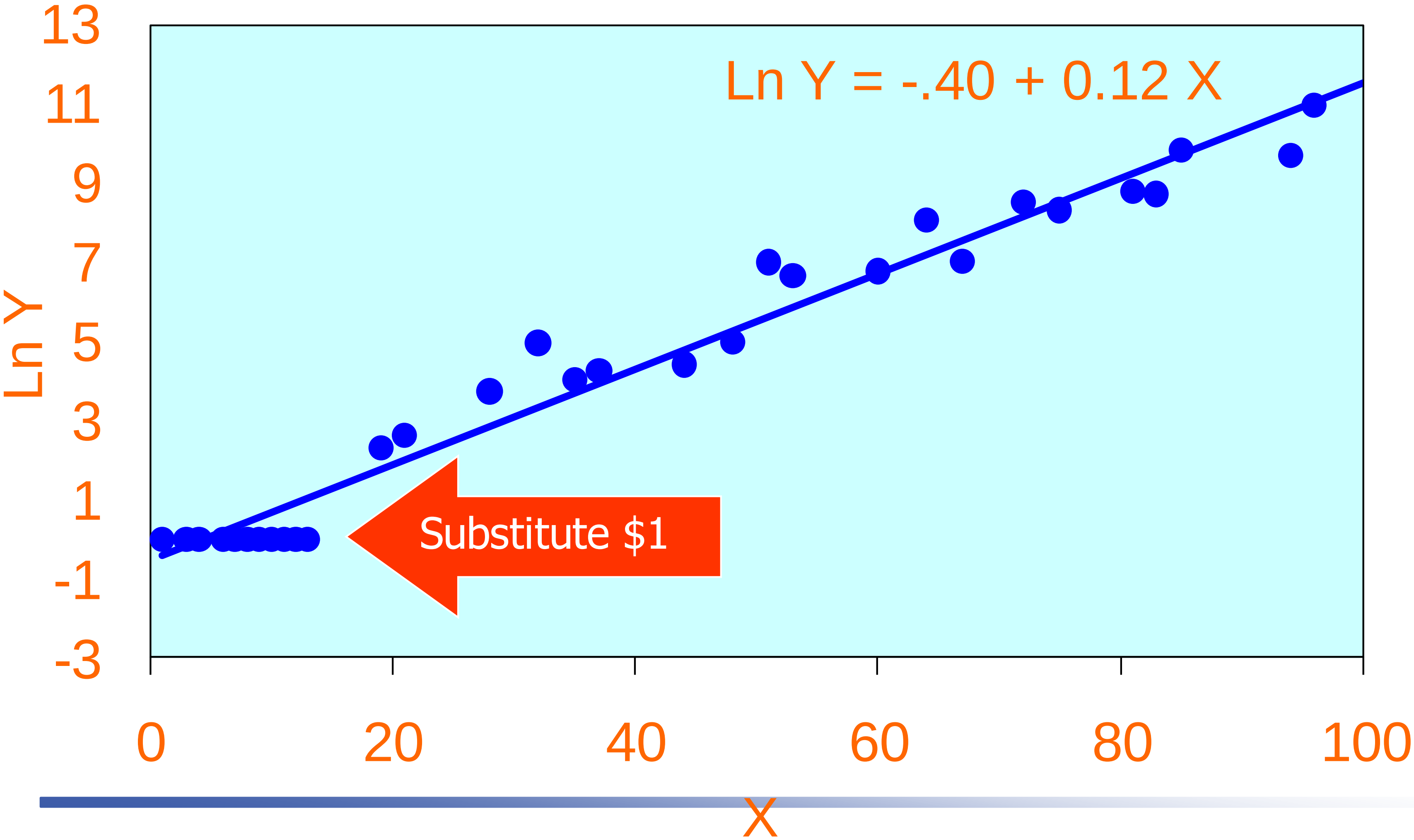
Zero values in cost data

- The other problem: left edge of distribution is truncated by observations where no cost is incurred
- How can we find Ln(Y) when Y = 0?
- Recall that Ln (0) is undefined

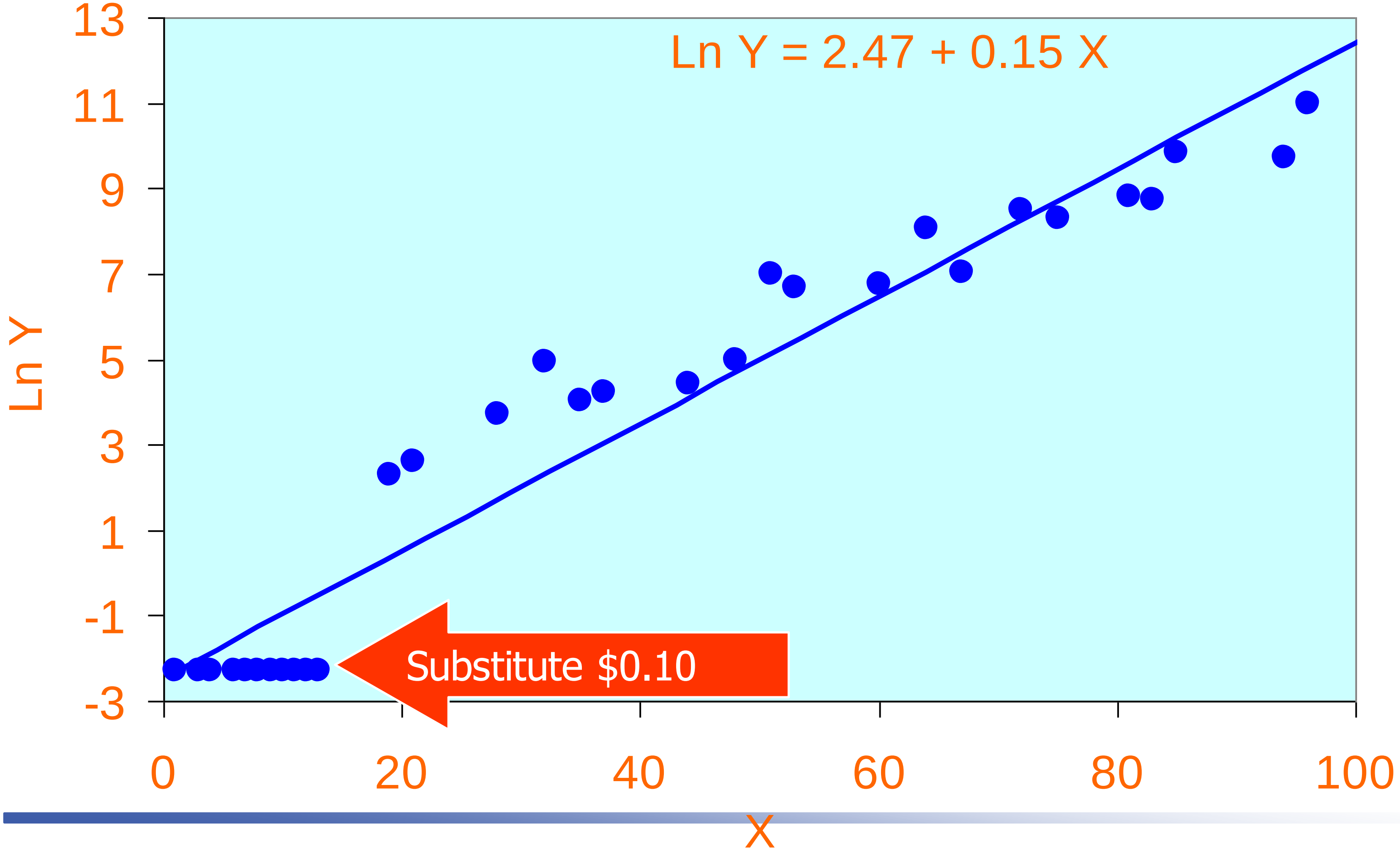
Log transformation

- Can we substitute a small positive number for zero cost records, and then take the log of cost?
  - $0.01, or $0.10, or $1.00?

Substitute $1 for Zero Cost Records



Substitute $0.10 for Zero Cost Records



Substitute small positive for zero cost?

- Log model assumes parameters are linear in logs
- Thus it assumes that change from $0.01 to $0.10 is the same as change from $1,000 to $10,000
- Possible to use a small positive number in place of zeros
  - if just a few zero cost records are involved
  - if results are not sensitive to choice of small positive value
- There are better methods!
  - Transformations that allows zeros (square root)
  - Two-part model
  - Other types of regressions

Is there any use for OLS with untransformed cost?

- OLS with untransformed cost can be used:
  - When costs are not very skewed
  - When there aren't too many zero observations
  - When there is large number of observations
- Parameters are much easier to explain
- Can estimate in a single regression even though some observations have zero costs
- The reviewers will probably want to be sure that you considered alternatives!

Review

- Cost data are not normal
  - They can be skewed (high cost outliers)
  - They can be truncated (zero values)
- Ordinary Least Squares (classical linear model) assumes error term (hence dependent variable) is normally distributed

Review

- Applying OLS to data that aren’t normal can result in biased parameters (outliers are too influential) especially in small to moderate sized samples

Review

- Log transformation can make cost more normally distributed so we can still use OLS
- Log transformation is not always necessary or the only method of dealing with skewed cost

Review

- Meaning of the parameters depends on the model
  - With linear dependent variable:
    - β is the change in *absolute units* of Y for a unit change in X
  - With logged dependent variable:
    - β is the *proportionate change* in Y for a unit change in X

Review

- To find fitted value a with linear dependent variable
- Find the linear combination of parameters and variables, e.g.

$$\hat{Y} \mid (X = 1, Z = \overline{Z}) = \alpha + \beta_1 + \beta_2 \overline{Z}$$

Review

- To find the fitted value with a logged dependent variable
- Can’t simply take anti-log of the linear combination of parameters and variables
- Must correct for retransformation bias

Review

- Retransformation bias can be corrected by multiplying the anti-log of the fitted value by the smearing estimator
- Smearing estimator is the mean of the antilog of the residuals

Review

- Cost data have observations with zero values, a truncated distribution
- Ln (0) is not defined
- It is sometimes possible to substitute small positive values for zero, but this can result in biased parameters
- There are better methods

Next session- June 3

- Two-part models
- Regressions with link functions
- Non-parametric statistical tests
- How to determine which method is best?

Reading assignment on cost models

Basic overview of methods of analyzing costs

- P Dier, D Yanez, A Ash, M Hornbrook, DY Lin. Methods for analyzing health care utilization and costs <u>Ann Rev Public Health</u> (1999) 20:125-144

- HERC@va.gov

Supplemental reading on Log Models

- Smearing estimator for retransformation of log models
  - Duan N. Smearing estimate: a nonparametric retransformation method. Journal of the American Statistical Association (1983) 78:605-610.
- Alternatives to smearing estimator
  - Manning WG. The logged dependent variable, heteroscedasticity, and the retransformation problem. Journal of Health Economics (1998) 17(3):283-295.

Appendix: Derivation of the meaning of the parameter in log model

$$\mathrm{Ln\ Y} = \alpha + \beta\ \mathrm{X} + \mu$$

$$\frac{dLnY}{dx} = \beta, as\ dLnY = dY / Y$$

$$\frac{dY / Y}{dx} = \beta$$

β is the proportional change in Y for a small change in X