

# Propensity Scores

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# Learning Objectives

- We will:
  - Define a propensity score
  - Identify methods for implementing a propensity score
  - Highlight the assumptions needed to make causal claims with observational data

# Outline

1. Background on assessing causation
2. Define propensity score (PS)
3. Calculate the PS
4. Use the PS
5. Limitations of the PS

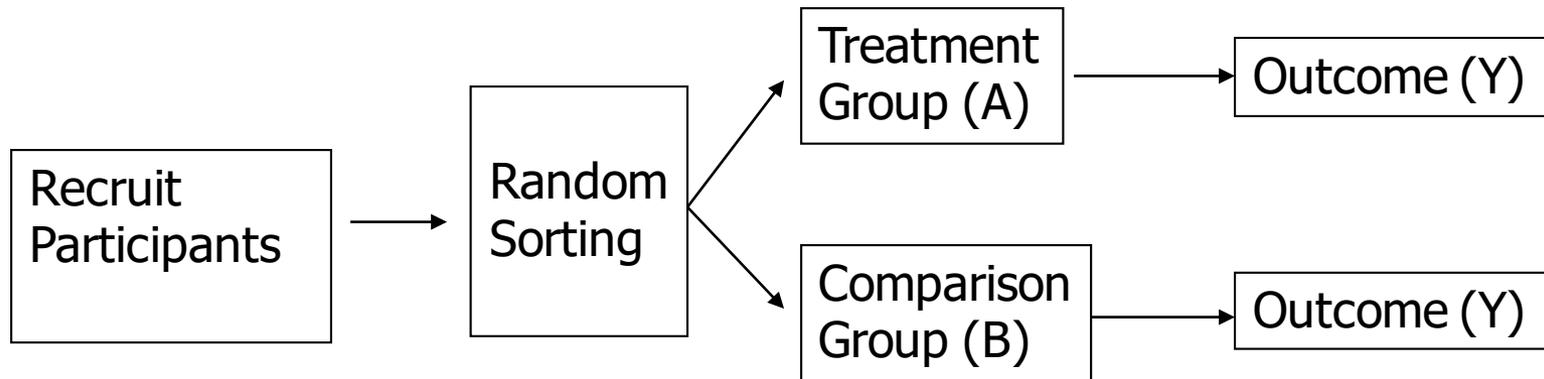
# Causality

- Researchers are often interested in understanding causal relationships
  - Does treatment X reduce symptoms?
  - Does volume of work affect job burnout?
  - Does the Veterans Crisis Line reduce the likelihood of suicide?
  - Are there drugs that people take that increase or decrease their risk of COVID-19.  
[<https://www.medrxiv.org/content/10.1101/2020.12.18.20248346v1>]

# Randomized Clinical Trial

- RCT provides a methodological approach for understanding causation
- Understanding propensity score is assisted by understanding randomized trials.

# Randomization



Note: random sorting can, by chance, lead to unbalanced groups. Most trials use checks and balances to preserve randomization

Just because a RCT can speak to causality, you must ask the question for whom— generalizability is often very limited

# Trial analysis

- The expected effect of treatment is

$$E(Y) = E(Y^A) - E(Y^B)$$

Expected effect on group A minus expected effect on group B (i.e., mean difference).

# Trial Analysis (II)

- $E(Y) = E(Y^A) - E(Y^B)$  can be analyzed using the following general model

$$y_i = \alpha + \beta x_i + \varepsilon_i$$

Where

- $y$  is the outcome
- $\alpha$  is the intercept
- $x$  is the mean difference in the outcome between treatment A relative to treatment B
- $\varepsilon$  is the error term
- $i$  denotes the unit of analysis (person)

# Trial Analysis (III)

- The model can be expanded to control for baseline characteristics

$$y_i = \alpha + \beta x_i + \delta Z_i + \varepsilon_i$$

Where

- $y$  is outcome
- $\alpha$  is the intercept
- $x$  is the added value of the treatment A relative to treatment B
- $Z$  is a vector of baseline characteristics (predetermined prior to randomization)
- $\varepsilon$  is the error term
- $i$  denotes the unit of analysis (person)

# Assumptions Needed for Causality

$$y_i = \alpha + \beta x_i + \varepsilon_i$$

- **X**, our right-hand side variable of interest, is measured without noise
  - Considered fixed in repeated samples
  - Noise, if it exists, is random, doesn't affect the mean, and biases towards the null
- There is no correlation between the **X** and the error term
  - In a RCT, this should happen by construction (coin flip) [ $E(x_i \varepsilon_i) = 0$ ]
  - Still must test balance of coin flip
- If these conditions hold,  $\beta$  on the treatment assignment is an unbiased estimate of the **causal** effect of **X** on the outcome

# What if...

- The assumptions don't hold in an RCT. Then what?
- You lose the unbiased estimate of causality.

# Observational Studies

- Randomized trials may be
  - Unethical
  - Infeasible
  - Impractical
  - Not scientifically justified

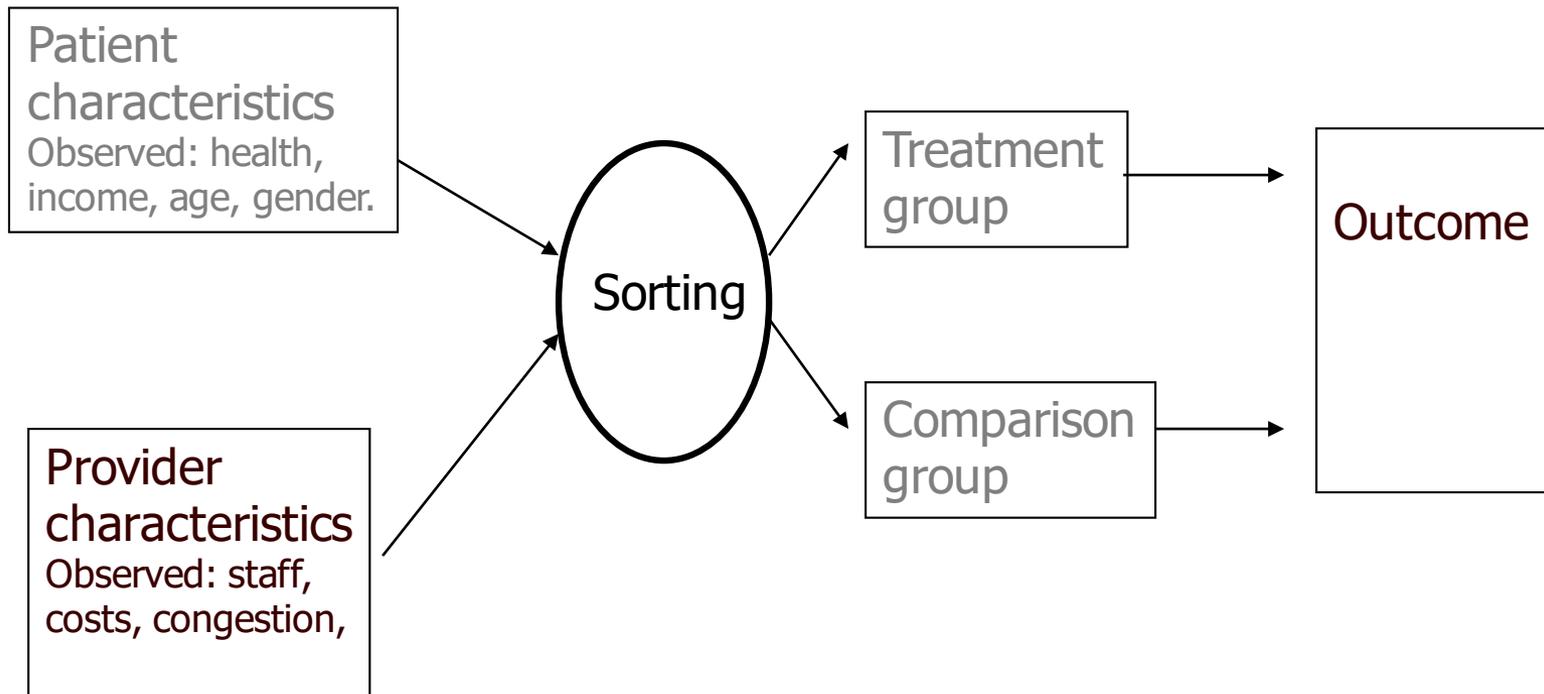
# Endogenous

- Not attributable to any external factor.
- Example: Does smoking lead to cancer

$$\text{cancer}_i = \alpha + \beta \text{smoking}_i + \varepsilon_i$$

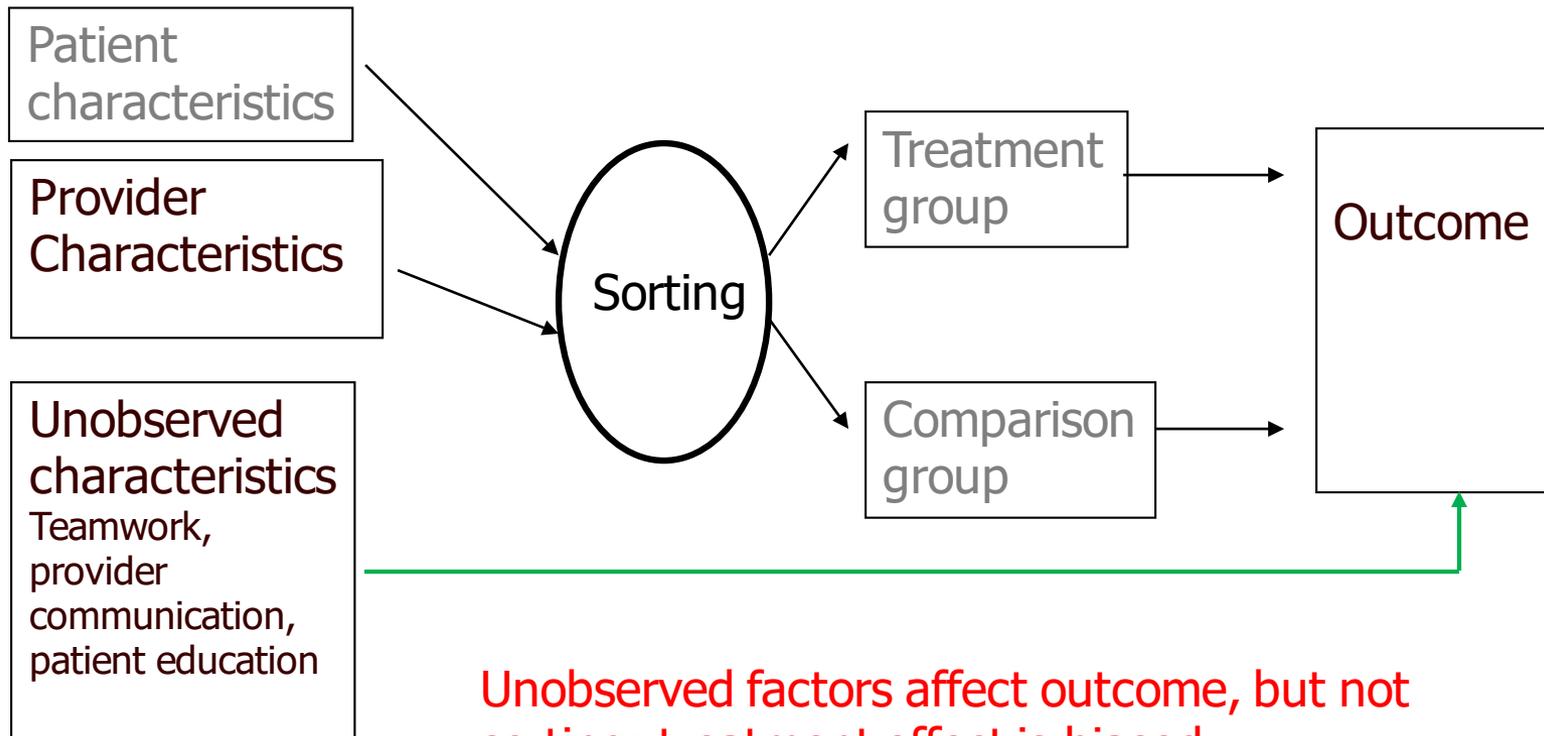
- Smoking is correlated with income, education, parental exposure, etc.
- We aren't controlling for any of those factors, thus  $E(\text{smoking}_i, \varepsilon_i) \neq 0$
- Thus, smoking is endogenous

# Sorting without randomization



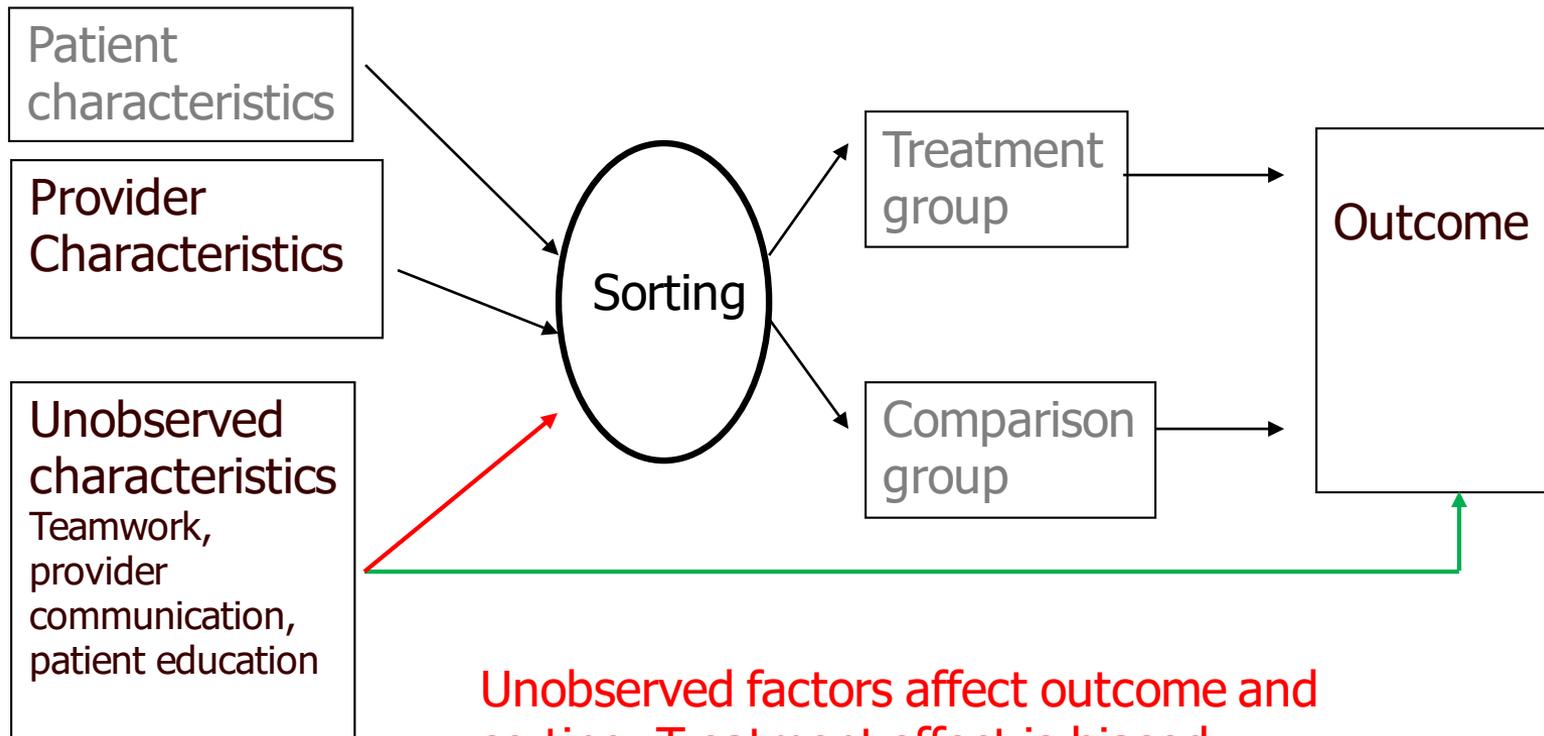
If everything is fully observed and correctly specified;  
results are not biased. **Never happens in reality.**

# Sorting without randomization



Unobserved factors affect outcome, but not sorting; treatment effect is biased.  
Fixed effects would be potential fix.

# Sorting without randomization



Unobserved factors affect outcome and sorting. Treatment effect is biased. Causality isn't identified.

# Example: Residential Treatment Programs

FIGURE 1 Unadjusted Average Daily Costs for Inpatient Psychiatry ( $N = 141$ )

Note: RTP = rehabilitation treatment program.

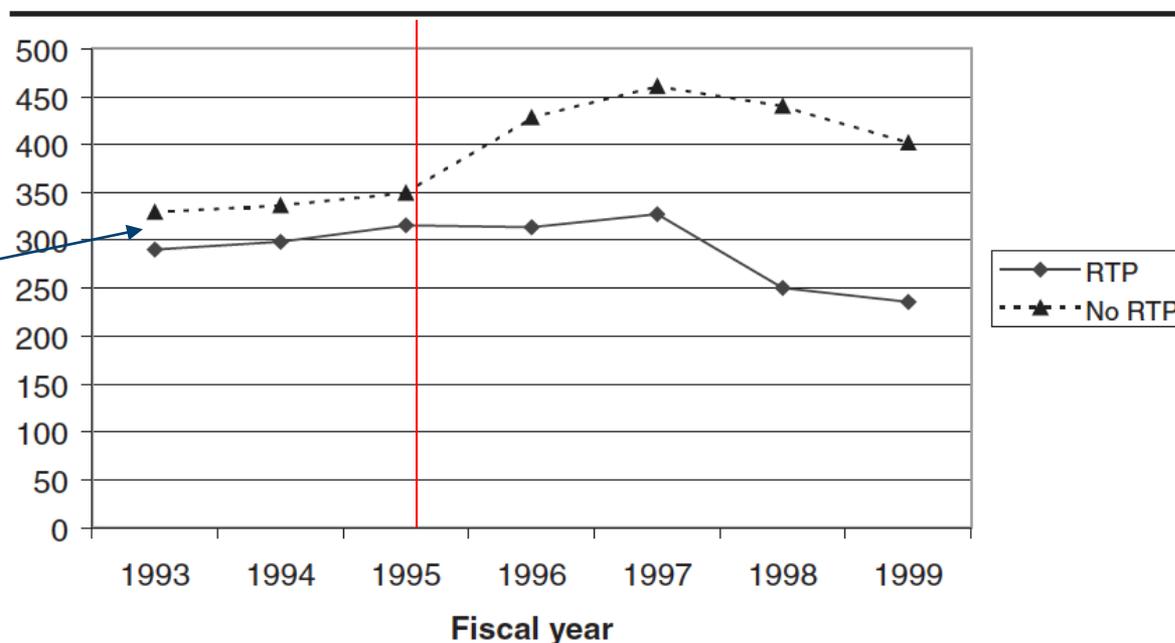


FIGURE 2 Unadjusted Average Daily Costs for Inpatient Substance Use ( $N = 134$ )

Note: RTP = rehabilitation treatment program.

Fixed effect removes level effect. Still assumes exogeneity

# Propensity Score Defined

- The PS uses observed information to calculate a single variable (the score)
- The score is the predicted propensity to get sorted (usually thought of as propensity to get treatment).

Expected treatment effect:  $E(Y) = E(Y^A) - E(Y^B)$

Propensity Score is:  $\Pr(Y=A \mid X_i)$

# Propensity Scores

- What it is: Another way to correct for observable characteristics
- What it is not: A way to adjust for unobserved characteristics
- The only way to make causal claims is to make **huge** assumptions.

# Strong Ignorability / Unconfounded

- To make statements about causation, you would need to make an assumption that treatment assignment is strongly ignorable.
  - Similar to assumptions of missing at random
  - Equivalent to stating that all variables of interest are observed
- Growing interest in using propensity scores for prediction, which is a separate issue

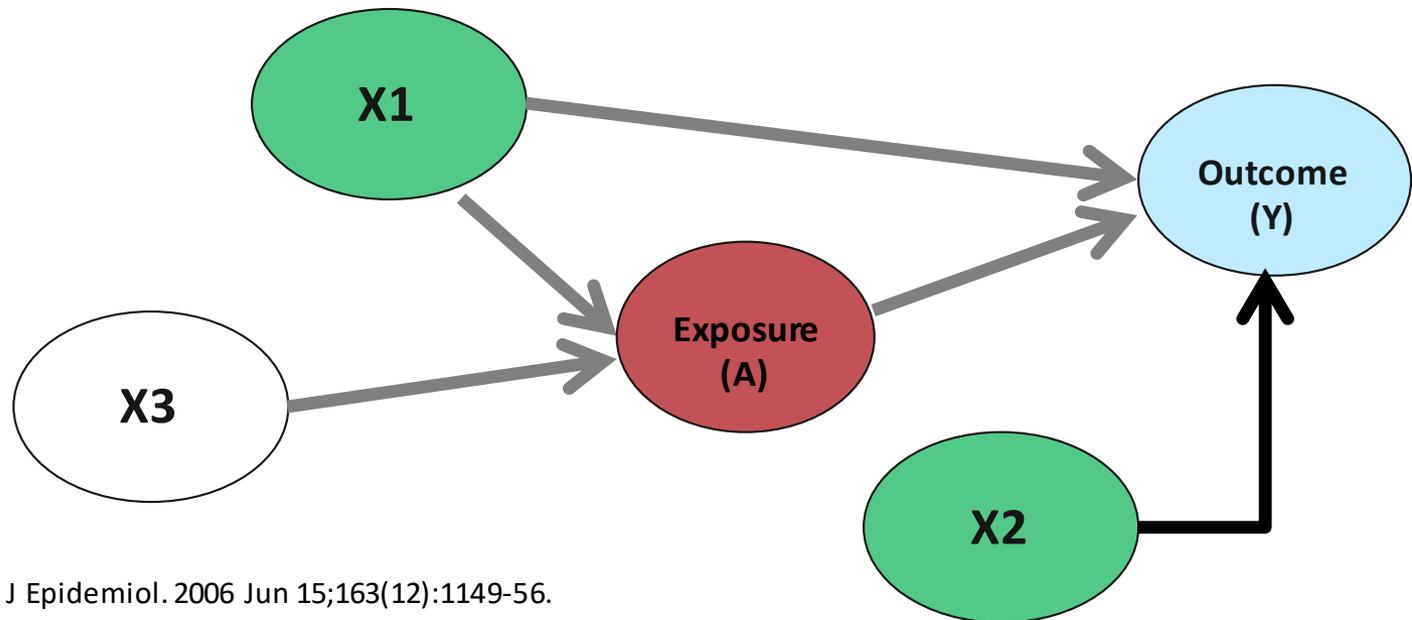
# Creating a Propensity Score

# Calculating the Propensity Score

- You observe key covariate of interest  
 $\text{cancer}_i = \alpha + \beta \text{smoking}_i + \varepsilon_i$
- Use multivariate logistic regression to estimate the probability that a person smoked
- The predicted probability from the logistic model is the propensity score
- PS models focus on sort into 2 groups, Melissa Garrido will be presenting later this year on 3-group PS models

# Variables to Include

- Include variables that are related to the observed outcome
- This will decrease the variance of an estimated exposure effect without increasing bias
- Do not include variables affect only correlated with exposure



# Variables to Exclude

- Exclude variables that are related to the exposure but not to the outcome
- These variables will increase the variance of the estimated exposure effect without decreasing bias
- Variable selection is particularly important in small studies ( $n < 500$ )

# Example: Resident Surgery

- Are patient outcomes different when the surgery is conducted by a resident or an attending?
- We had a dataset that tracked the primary surgeon for heart bypass

# Uses

- Understanding sorting and balance
  - Sorting is multidimensional
  - The PS provides a simple way of reducing this dimensionality to understand the similarity of the treatment groups
- Adjusting for covariance

# Example

- Are surgical outcomes worse when the surgeon is a resident?
- Resident assignment may depend on
  - Patient risk
  - Availability of resident
  - Resident skill
  - Local culture

# Resident Assignment

	OR	P value
Age	1.00	0.79
Canadian Functional Class		
Class 2	1.93	0.15
Class 3	2.12	0.09
Class 4	4.25	0.02
Urgent priority	0.93	0.89
Artery condition at site		
Calcified	0.67	0.25
Sclerotic	2.63	0.00
site 2	62.89	0.00
site 3	0.67	0.60
site 5	138.16	0.00
site 7	11.66	0.00
site 8	19.85	0.00
site 9	1.76	0.43
endo vascular harvest	0.20	0.01
On pump surgery	1.20	0.75
1-2 grafts	1.70	0.16
4-5 grafts	0.79	0.46

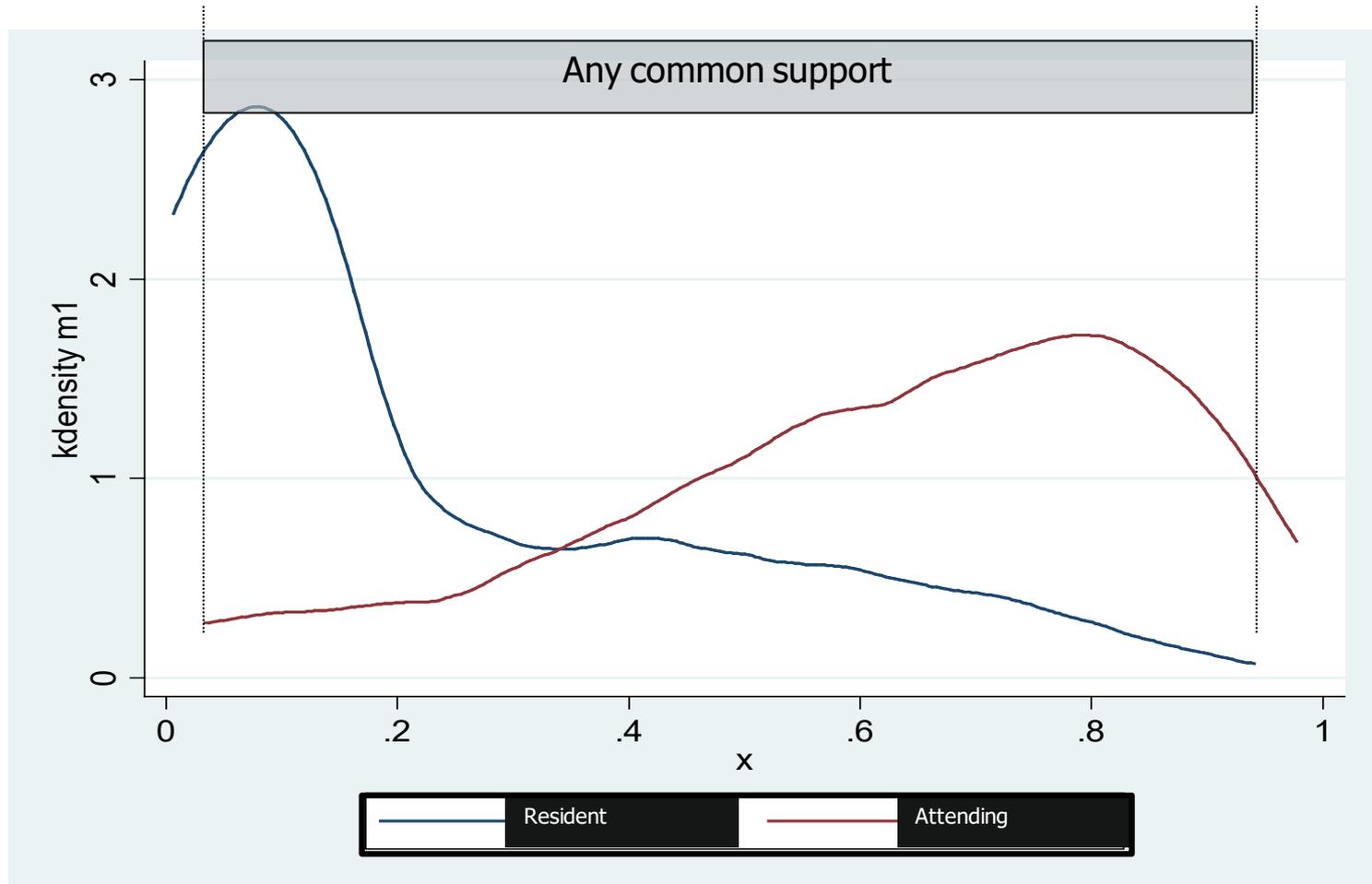
Assignment not associated with age or number of grafts

Assignment associated with angina symptoms and planned harvesting technique

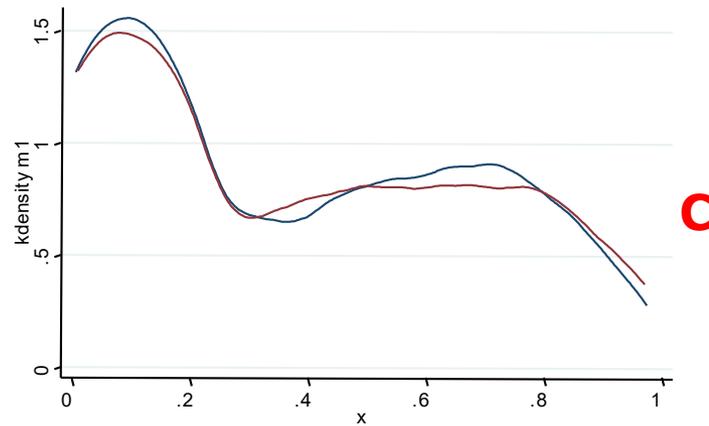
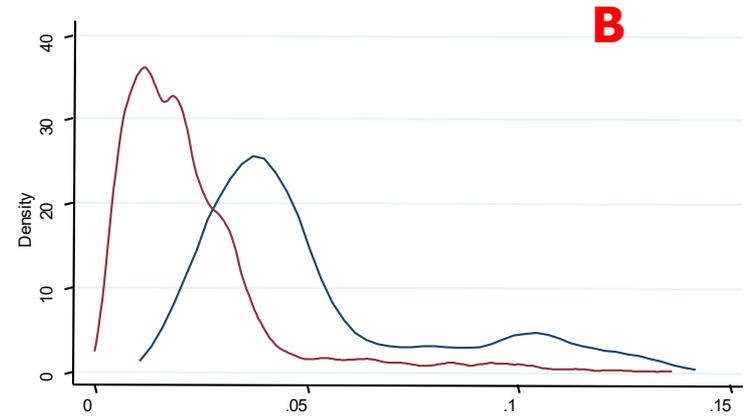
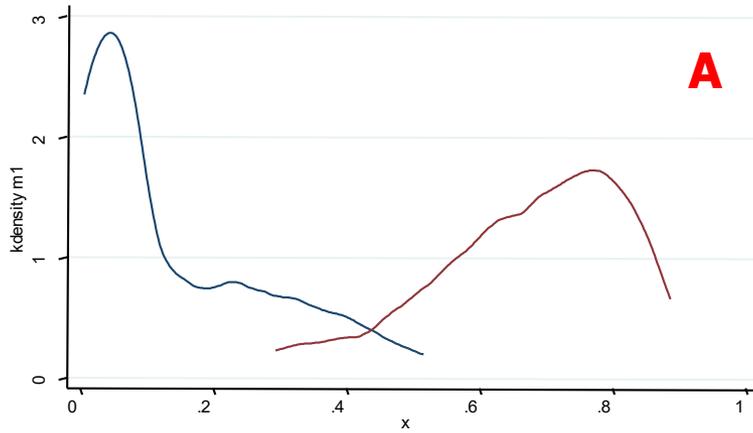
# Shared / Common Support

- Measures the similarity of people in both treatments
- Conditional on covariates, there exist people who choose both treatments.
- Examining shared support offers insights not in multivariate models 

# Propensity Score for Resident vs Attending Surgeon



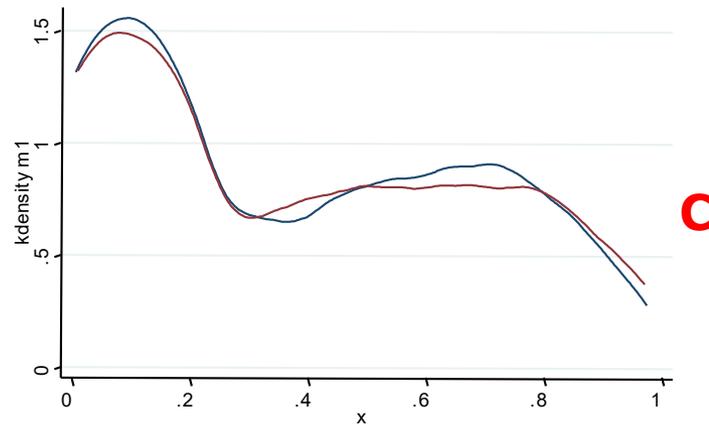
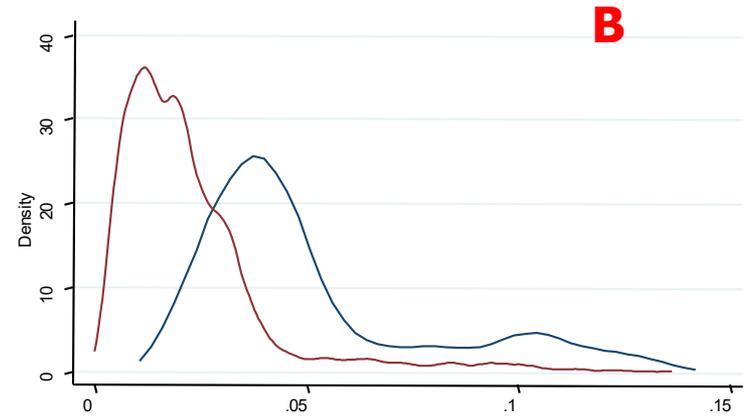
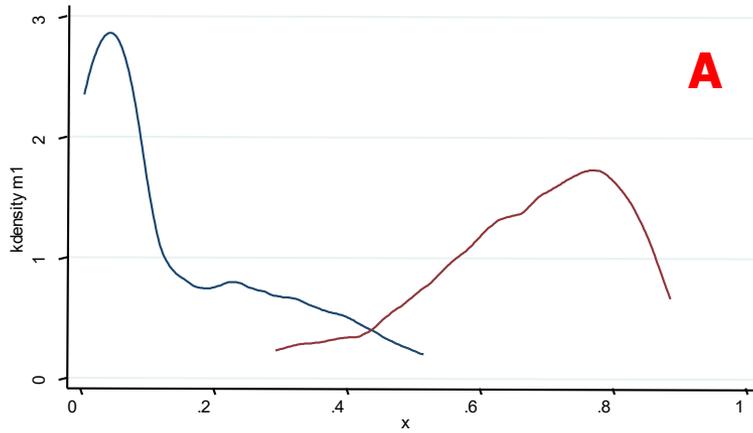
# Compare Three Diagrams



# Poll

- Which graph is the most concerning?  
Choose one
  - A
  - B
  - C
  - All of them
  - None of them

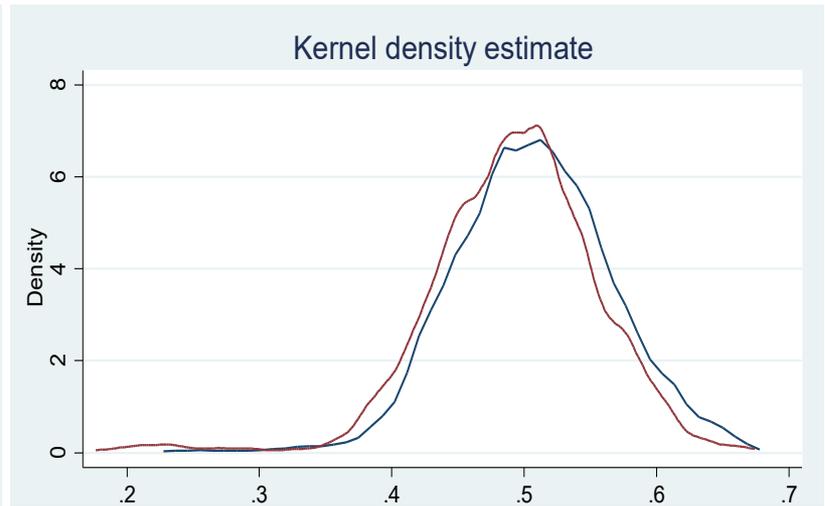
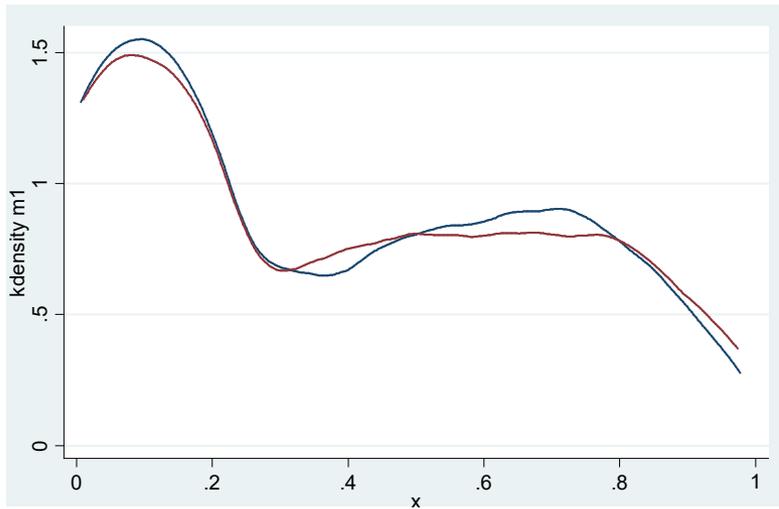
# Three Scores



# RCTs and Propensity Scores

- What would happen if you used a propensity score with data from a RCT?

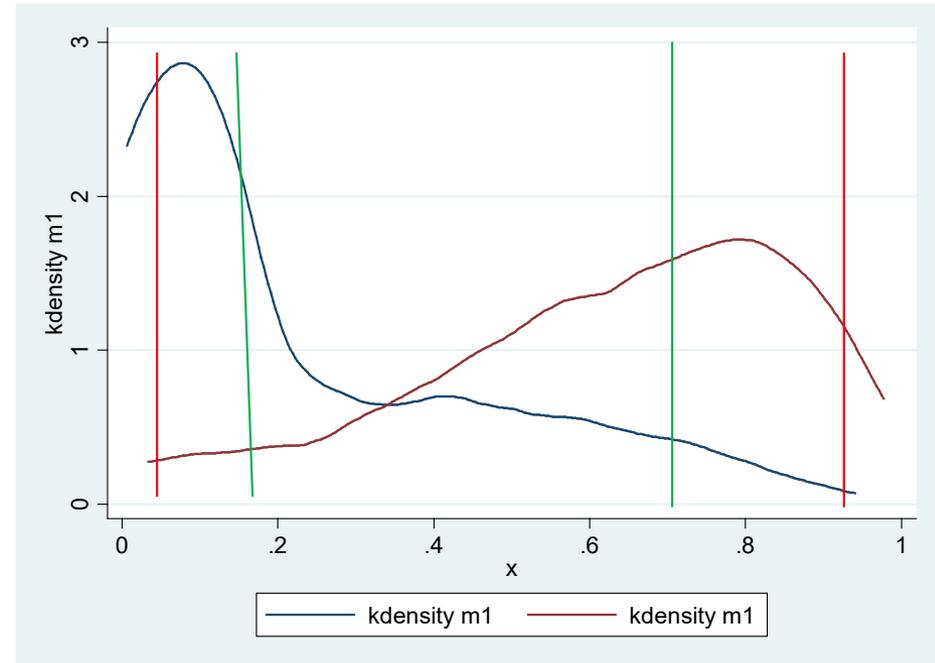
# Shared Common Support



Don't worry about the shape. Focus on the overlap

# Common Support

- Understanding the shared support is critical
  - What do you do with observations that don't share support?
  - Where do you draw the line?
  - What do you do with observations outside the bounds?



# Using the Propensity Score

# Using the Propensity Score

1. Compare individuals based on similar PS scores (a matched analysis)
2. Conduct subgroup analyses on similar groups (stratification)
3. Include it as a covariate (quintiles of the PS) in the regression model
4. Use it to weight the regression (i.e., place more weight on similar cases)
5. Use both 3 and 4 together (doubly robust)

# PS as a Covariate

- There seems to be little advantage to using PS over multivariate analyses in most cases.<sup>1</sup>
- PS provides flexibility in the functional form
- Propensity scores may be preferable if the sample size is small and the outcome of interest is rare.<sup>2</sup>

1. Winkelmeier. Nephrol. Dial. Transplant 2004; 19(7): 1671-1673.

2. Cepeda et al. Am J Epidemiol 2003; 158: 280-287

# Matched Analyses

- The idea is to select controls that resemble the treatment group in all dimensions, except for treatment
- You can exclude cases and controls that don't match, which can reduce the sample size/power.
- Different matching methods

# Matching Methods

- Nearest Neighbor: rank the propensity score and choose control that is closest to case.
- Caliper: choose your common support and from within randomly draw controls
- Choice of matching estimator important

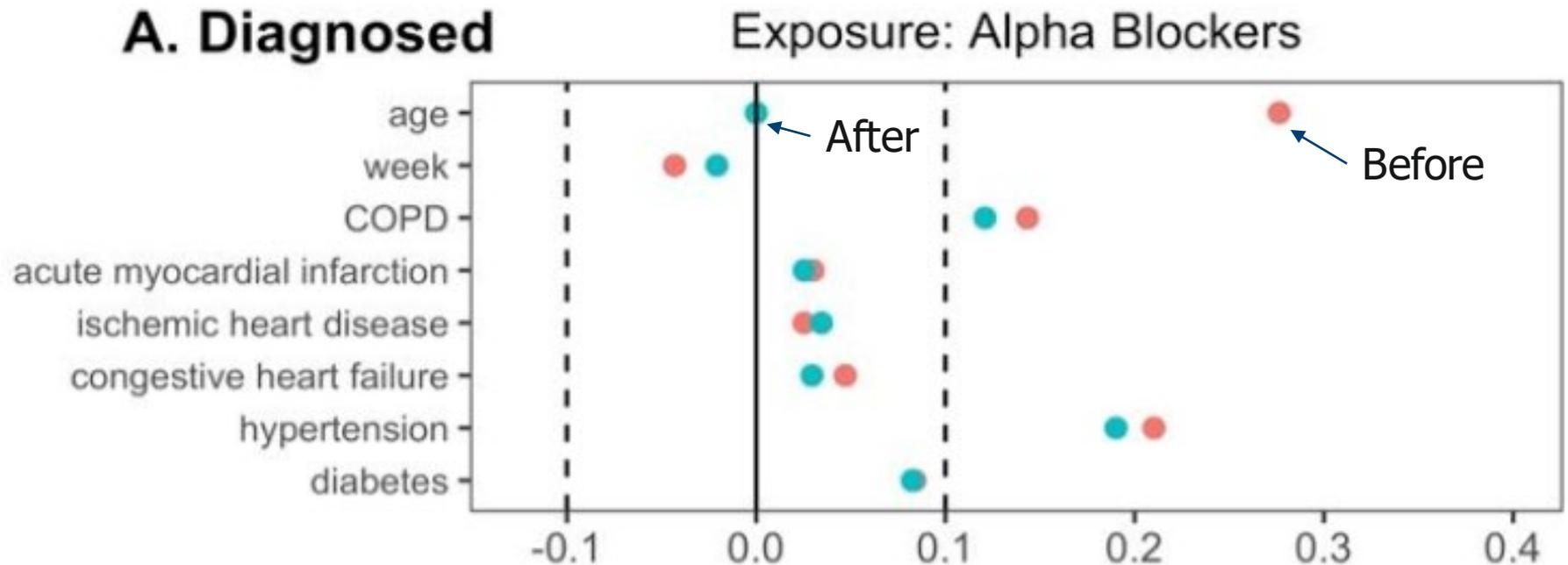
# Next Step

- Choose your method
- Graph the overlap
- Compare the balance (Love plots)
  - Standardized difference of less than 10% is a common rule of thumb

Attributed to Thomas E Love (Heart failure, chronic diuretic use, and increase in mortality and hospitalization. European Heart Journal. 2006 ).

# Love Plots

**The Association Between Alpha-1 Adrenergic Receptor Antagonists and In-Hospital Mortality from COVID-19**



# Recent Areas of Research

## ■ Economics: choice of matching estimators

- Busso M et al. New Evidence on the Finite Sample Properties of Propensity Score Reweighting and Matching Estimators. *Review of Economics and Statistics*, 96.5 (2014): 885-897
- Athey S, Imbens GW. The state of applied econometrics: Causality and policy evaluation. *Journal of Economic Perspectives*. 2017 May;31(2):3-2.

## ■ Political Science

- King G, Nielsen R. Why propensity scores should not be used for matching. Copy at <http://j.mp/1sexgVw>. 2016 Dec 16;378.

## ■ Biostatistics: high dimensional propensity scores using big data

- Schneeweiss, Sebastian, et al. "High-dimensional propensity score adjustment in studies of treatment effects using health care claims data." *Epidemiology* 20.4 (2009): 512.

# Limitations

# Do the Unobservables Matter?

- Propensity scores focus only on observed characteristics, not on unobserved.
- Improbable that we fully observe the sorting process
  - Thus  $E(x_i \varepsilon_i) \neq 0$
  - Multivariate (including propensity score) is biased and we need another method, such as instrumental variables, fixed effects or RCT

# Does Using PS Exacerbate Imbalance of Unobservables

- PS is based on observables.
- Brooks and Ohsfeldt, using simulated data, showed that PS models can create **greater** imbalance among unobserved variables.
- King G, Nielsen R. Why propensity scores should not be used for matching.  
<https://dspace.mit.edu/handle/1721.1/128459>

Brooks and Ohsfeldt (2013): Squeezing the balloon: propensity scores and unmeasured covariate balance. *Health Services Research*.

# Summary

# Overview

- Propensity scores offer another way to adjust for confound by observables
- Reducing the multidimensional nature of confounding can be helpful
- There are many ways to implement propensity scores and a growing interest in matching estimators

# Strengths

- Allow one to check for balance between control and treatment
- Without balance, average treatment effects can be very sensitive to the choice of the estimators.<sup>1</sup>

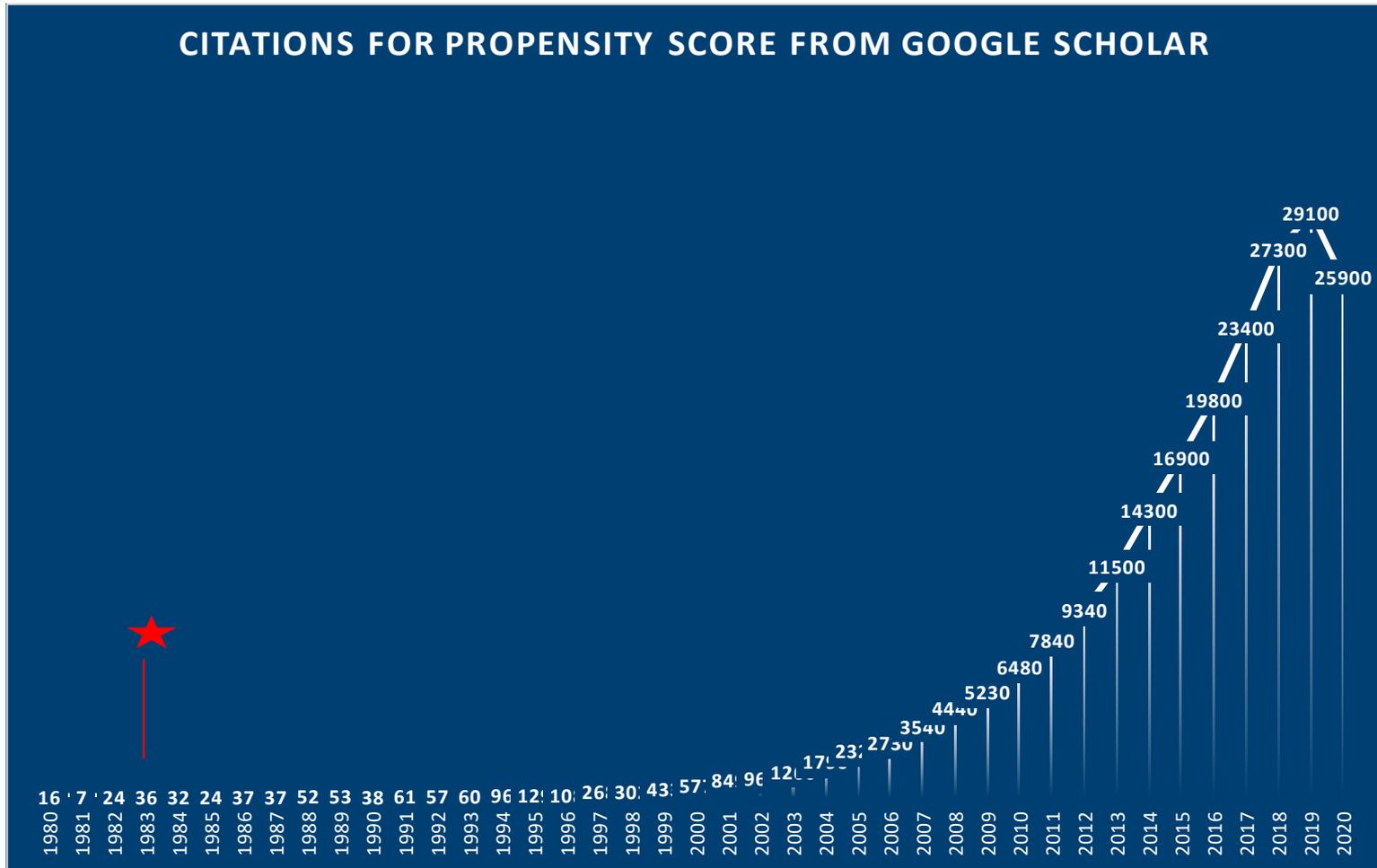
# Challenges

- Propensity scores are often misunderstood
- Not enough attention is placed on the PS model, itself
- Not enough attention is placed on robustness checks
- While a PS can help create balance on observables, PS models do not control for unobservables or selection bias

# Further Reading

- Rosenbaum, P. R., D. B. Rubin. "The central role of the propensity score in observational studies for causal effects." *Biometrika* 70 (1983): 41–55
- Imbens and Wooldridge (2007) [www.nber.org/WNE/lect\\_1\\_match\\_fig.pdf](http://www.nber.org/WNE/lect_1_match_fig.pdf)
- Imbens, Guido W. "The role of the propensity score in estimating dose-response functions." *Biometrika* 87.3 (2000): 706-710.
- Imbens, Guido W. "Nonparametric estimation of average treatment effects under exogeneity: A review." *Review of Economics and Statistics* 86.1 (2004): 4-29.
- Guo and Fraser (2010) *Propensity Score Analysis*. Sage.
- King G, Nielsen R. Why propensity scores should not be used for matching. Copy at <http://j.mp/1sexgVw>. 2016 Dec 16;378.
- Brooks, John M., and Robert L. Ohsfeldt. "Squeezing the balloon: propensity scores and unmeasured covariate balance." *Health Services Research* 48.4 (2013): 1487-1507.
- Garrido, Melissa M., et al. "Methods for constructing and assessing propensity scores." *Health Services Research* 49.5 (2014): 1701-1720.
- Busso M et al. "New Evidence on the Finite Sample Properties of Propensity Score Reweighting and Matching Estimators." *Review of Economics and Statistics*, 96.5 (2014): 885-897
- Imai, Kosuke, and Marc Ratkovic. "Covariate balancing propensity score." *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* 76.1 (2014): 243-263.

# The Future ?



# Questions?

- [HERC@VA.gov](mailto:HERC@VA.gov)

- [@herc\\_va](https://twitter.com/herc_va)

- [@toddhwagner](https://twitter.com/toddhwagner) 

- **Next class: Natural Experiments and Difference-in-Differences**

Jean Yoon, Ph.D. Feb 10.